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TFaNS Tone Fan Noise Design/Prediction System

Volume I: System Description, CUP3D Technical Documentation and Manual for Code Developers

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FOREWORD

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SUMMARY

TFaNS is the Tone Fan Noise Design/Prediction System developed by Pratt & Whitney under contract to NASA Lewis. The purpose of this system is to predict tone noise emanating from a fan stage including the effects of reflection and transmission by the rotor and stator and by the duct inlet and nozzle. These effects have been added to an existing annular duct/isolated stator noise prediction capability.

The underlying concept for the system was presented in Reference 1 with application to cascades in 2–dimensional channels. TFaNS extends this to annular geometry with duct terminations and radiation to the far–field via the following scheme: "Acoustic elements" (e.g. the inlet, the rotor, the stator, and the nozzle) are first analyzed in isolation to determine their modal reflection and transmission coefficients (including frequency scattering in the case of the rotor). Then the elements are coupled as a linear system via the duct eigenmodes at "interface planes" separating the elements. The linear system is solved to find a "state vector" of mode amplitudes at the interface planes. The "state vector" then is used to compute upstream and downstream modal sound powers and the sound pressure directivities in the outside field.

TFaNS consists of:

- The codes that compute the acoustic properties (reflection and transmission coefficients) of the various elements and writes them to Acoustic Properties Files,
- CUP3D: Fan Noise Coupling Code that reads these files, solves the coupling equations, and outputs the desired noise predictions,
- AWAKEN: CFD/Measured Wake Postprocessor which reformats CFD wake predictions and/or measured wake data so they can be used by the system to predict noise.

Acoustic properties can be computed from a variety of codes other than those presently in TFaNS. For example, rotor and stator reflection and transmission are currently being computed from a classical, uniform axial flow model with solid body swirl. However, more sophisticated codes (e.g. linearized Euler codes) are anticipated for future application. CUP3D has been written in a general form so that acoustic properties from a variety of codes can be handled. To accomplish this, a standard file format for acoustic properties has been developed and must be followed.

This document provides technical background for TFaNS including the organization of the system and CUP3D technical documentation. This document also provides information for code developers who must write Acoustic Properties Files in the CUP3D format.

As described on page 3, technical documentation of the other TFaNS codes may be found in References 2 to 7. The TFaNS user's manual may be found in Reference 8. Evaluation of the system may be found in Reference 9.

1. INTRODUCTION

The TFaNS coupling and noise prediction scheme is explained conceptually with reference to Figure 1.

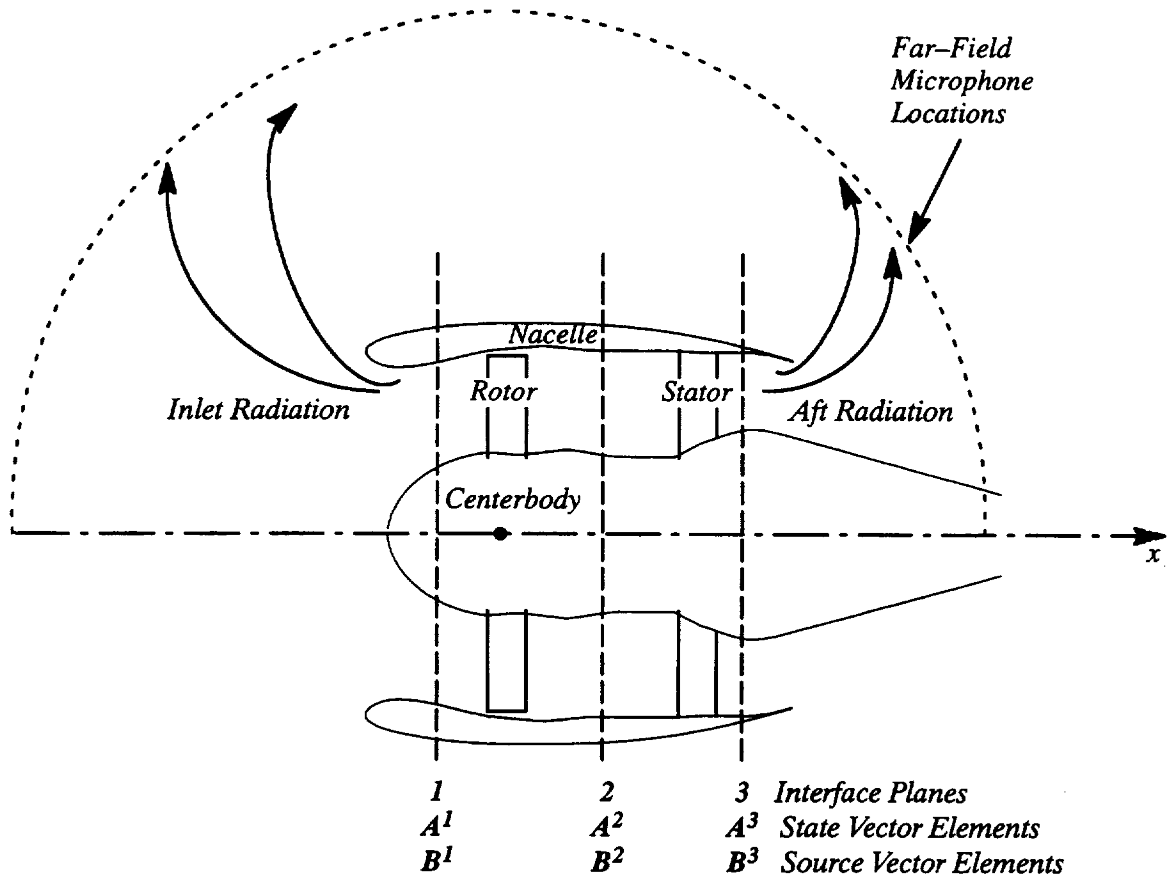

Figure 1: TFaNS Coupling and Noise Prediction Scheme

The fan stage is divided by three interface planes into four acoustic elements (inlet, rotor, stator, and nozzle). State vector, $\boldsymbol{A}$, is in three sections, $\boldsymbol{A}^1$, $\boldsymbol{A}^2$, and $\boldsymbol{A}^3$, whose elements are modal amplitudes that will be defined in a later section. Source vector, $\boldsymbol{B}$, has the same structure. $\boldsymbol{B}$ is prescribed (corresponding, for example, to the output of a stator due to rotor wake input in an uncoupled environment) and $\boldsymbol{A}$ is to be found as a function of $\boldsymbol{B}$ by solving the linear system equations.

Coupling of the elements at the interface planes is specified in terms of the scattering matrix, $\boldsymbol{S}$, which is built up from modal reflection and transmission coefficients. In condensed form, the system equations are represented by

$$\boldsymbol{A} = \boldsymbol{S}\boldsymbol{A} + \boldsymbol{B} \tag{1}$$

which is to say that the state vector elements are the sum of the parts from scattering, $\boldsymbol{SA}$, and directly from the source, $\boldsymbol{B}$. These equations are solved formally by

$$\boldsymbol{A} = (\boldsymbol{1} - \boldsymbol{S})^{-1}\boldsymbol{B} \tag{2}$$

Two major forms of noise output are computed from the state vector:

- Upstream and downstream propagating sound power level are computed from A on a mode–by–mode basis. Power is calculated just upstream and just downstream of the noise source (defined in Figure 1 by interface plane 1 upstream, and interface plane 3 downstream).
- Outside far–field directivity is computed from the elements of A. In this case, far–field directivity shapes are computed by the radiation codes with unit amplitude input and stored as part of the acoustic properties files. These directivities are then multiplied by A to get the far–field sound pressure level directivity.

TFaNS (Version 1.4) consists of the following computer codes:

- CUP3D: Fan Noise Coupling Code Version 2.1 (Section 3., Appendices I to IV)
- SOURCE3D: Rotor Wake/Stator Interaction Code Version 2.5 (Reference 2)
- Eversman Inlet Radiation code Version 3.0 (References 3, 4, 5 and Appendix V)
- Eversman Aft Radiation code Version 3.1 (References 4, 5, 6 and Appendix V)
- AWAKEN: CFD/Measured Wake Postprocessor Version 1.0 (Reference 7)

The following sections present the organization of TFaNS, the technical background behind the CUP3D code, and the information necessary to write an Acoustic Properties File. The first part of the document discusses the TFaNS organization. Then the technical documentation for CUP3D is presented. This includes background on the coupling system, state vector conventions, definitions of the reflection and transmission coefficients (scattering coefficients) in terms of the state vector, and the calculation of far–field directivities and mode power levels. Appendix I presents the information necessary to create Acoustic Properties Files. Changes made to the Eversman inlet and aft radiation codes to make them part of TFaNS are also presented in Appendix V.

2. GENERAL ORGANIZATION OF TFaNS

The organization of TFaNS is illustrated in Figure 2. This figure identifies the codes which comprise the system and how they interact with each other.

The central portion of the system is the CUP3D Fan Noise Coupling Code. This code reads Acoustic Properties Files which contain scattering (transmission and reflection) coefficients from other codes along with far–field directivity shapes and source vector information (e.g. noise from rotor wake/stator interaction). This information is used to form a system of linear equations which permit acoustic elements to reflect and transmit to each other. A System File is also input which determines the organization of the acoustic elements. Output from this code includes far–field directivities along with inlet and aft power levels.

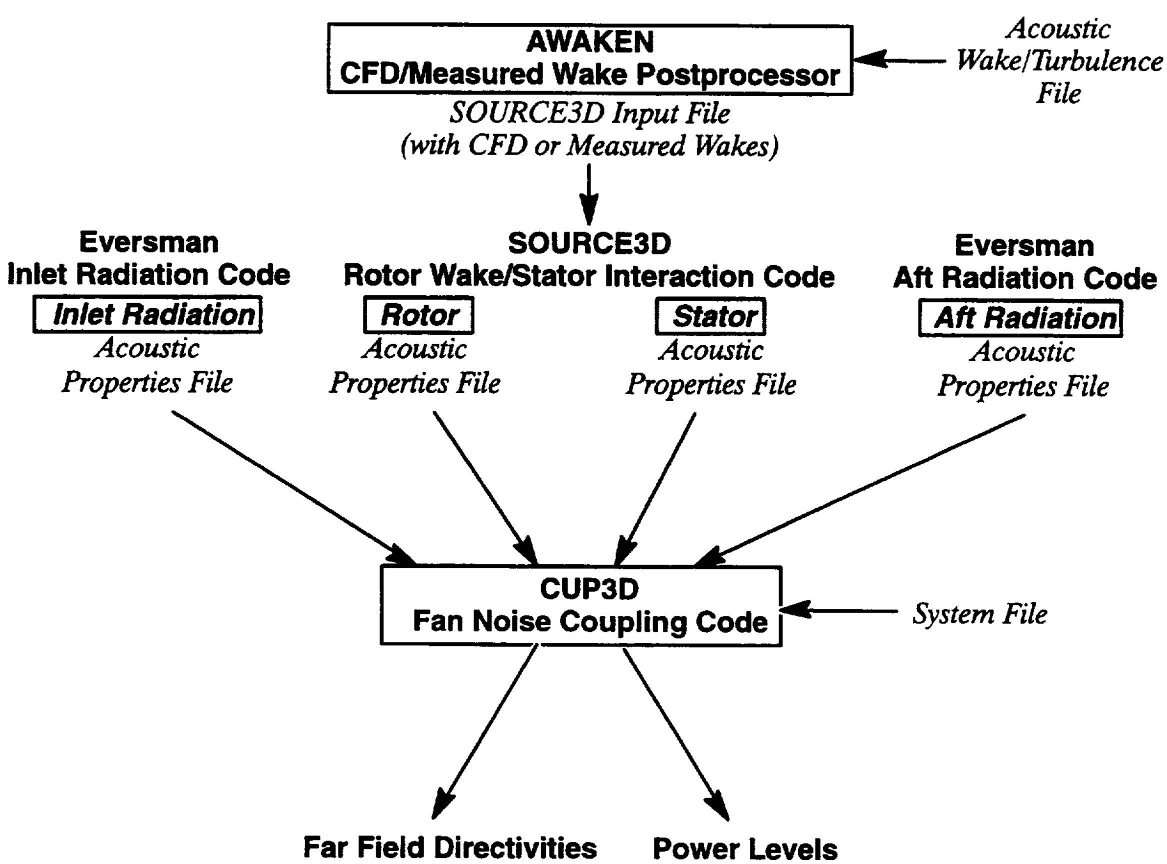

Figure 2: Organization of TFaNS Version 1.4

The SOURCE3D Rotor Wake/Stator Interaction Code is a significantly extended and improved version of the V072 Rotor Wake/Stator Interaction Code (Reference 10, 11 and 12). It has two functions within TFaNS: firstly, it calculates tone noise from a rotor wake/FEGV interaction and, secondly, it determines the scattering coefficients for the rotor and stator then outputs them to rotor and stator acoustic properties files (Figure 2) for use by CUP3D. This code can either use its own internal semi–empirical wake model, or it can use CFD or measured wakes processed through the AWAKEN CFD/Measured Wake Postprocessor.

The AWAKEN CFD/Measured Wake Postprocessor creates a SOURCE3D input file which contains upwash wake harmonic amplitudes calculated from CFD predictions or measured velocity data. CFD or measured velocity information is obtained from the Acoustic Wake/Turbulence File which is generated either by a CFD code (or postprocessor) or during a engine/rig test program.

The Eversman inlet and aft radiation codes are run if far−field directivities are required. These codes comprise three "modules". The first module creates a finite element mesh for the calculation. The second module calculates the potential steady flow field. Finally a radiation module, modified to interface with TFaNS (see Appendix V), is used to complete the potential steady flow calculation for a given duct flow condition and calculate the far field radiation and scattering coefficients for a specified number of blade passing frequency (BPF) harmonics on a mode−by−mode basis.

3. CUP3D FAN NOISE COUPLING CODE

3.1 GENERAL ORGANIZATION OF CUP3D

The CUP3D Fan Noise Coupling Code is the final link in the TFaNS Tone Fan Noise design/prediction System. Output files from isolated elements are used by CUP3D to "couple" the elements thus accounting for reflection and transmission of acoustic and vorticity waves throughout the system.

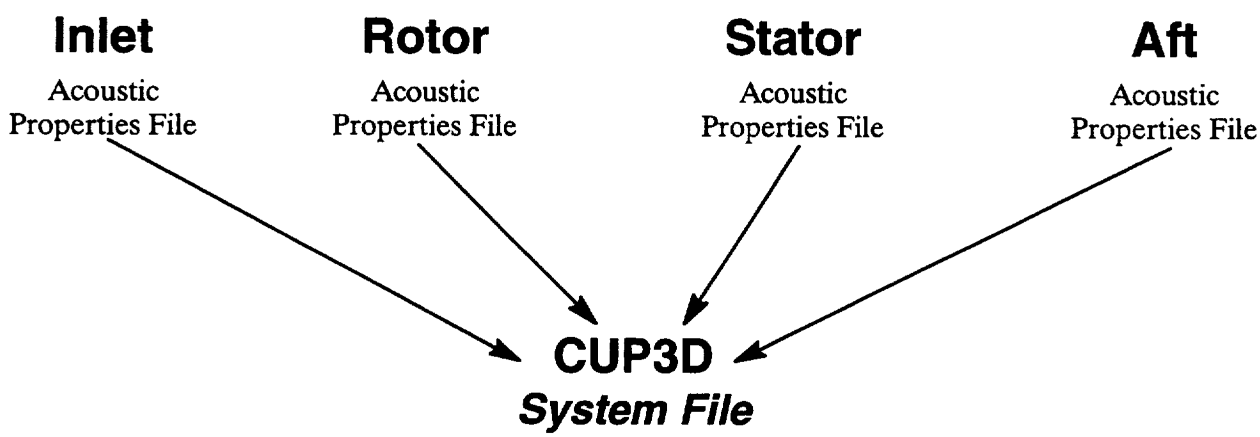

Figure 3: CUP3D File Organization

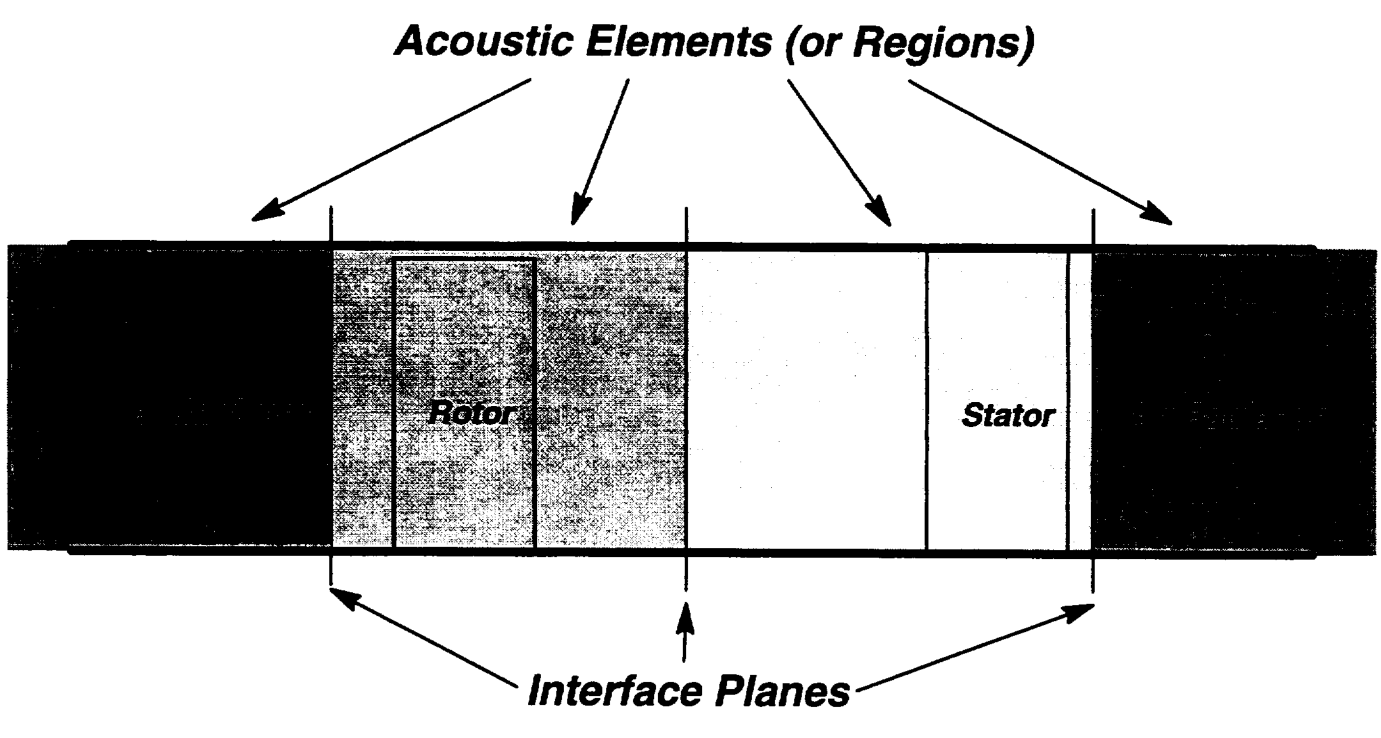

Figure 4: CUP3D Geometry and Terminology

Figure 3 is a block diagram showing the files required by CUP3D. First, codes such as an inlet radiation code, rotor code, stator code, and aft radiation code are run. These codes create Acoustic Properties Files which contain all the acoustic information about an acoustic element. The information may include geometry and performance, harmonic and mode numbers, scattering coefficients, source vector coefficients and far–field directivity shapes. Acoustic Properties Files can be output for any rotor, stator or radiation code. These Acoustic Properties Files and a user generated System File are then input into CUP3D. The System File contains information on how the various acoustic elements will be organized to form the total acoustic system. An example of an acoustic system is shown in Figure 4.

Information about the terminology and geometry used by CUP3D is given in Figure 4. The present system can be organized with up to four acoustic elements (or regions). These elements are classified by any of four types: inlet radiation, rotor, stator, and aft radiation. The number and types of elements used may vary from case to case. However, if radiation elements are not used, they must be replaced by non–reflecting regions which act to propagate the noise away from the system. Output from CUP3D includes inlet and aft mode and total power levels and far–field directivities.

Each acoustic element is bounded by interface planes. Rotor and stator acoustic elements are bounded by two interface planes. Inlet and aft radiation acoustic elements are bounded by one interface plane and the far–field. These boundaries are non–reflecting and permit waves to be sent into an acoustic element one at a time. Reflection and transmission scattering coefficients can then be calculated.

3.2 STATE VECTOR AND COUPLING COEFFICIENT CONVENTIONS

Reference 1 discusses in detail the physics of mode scattering in two dimensional flow. The purpose of this section is to present the equations needed to create scattering coefficients and source vector coefficients for the CUP3D code which handles information in three dimensions. This section is presented for the benefit of the source or radiation code developer who may need to create an Acoustic Properties File.

"Coupling coefficient" is the collective term which refers to reflection and transmission coefficients. These are the matrix elements in the scattering matrix, S, that specify how any mode input to an acoustic element is transmitted and reflected and scatters into other modes. Since these coefficients are defined in terms of state vector elements, or modal amplitudes, the modal representation is specified first in this section. Then the coupling coefficients are defined in terms of them.

Figure 5 shows the coordinate systems for CUP3D. Inside the engine the coordinate system is a cylindrical (r, ϕ, x) right handed system where x is positive in the downstream direction. The far–field uses a polar coordinate system (R, ϕ, θ) where θ is zero in front of the inlet.

As in Reference 1, the flow perturbations are represented by a series of pressure and vorticity waves. In general, the wave types, W, which are scattered by an acoustic element are affected by the type of flow through the duct. CUP3D uses a general set of modes based on whatever flow conditions are assumed at the interface planes (e.g. Reference 14). Though much of this report is written assuming a specific set of modes (given below), CUP3D is not limited to this set of modes. One exception: Sound power in CUP3D is presently computed using a specific set of acoustic modes. This calculation may be generalized in later code versions.

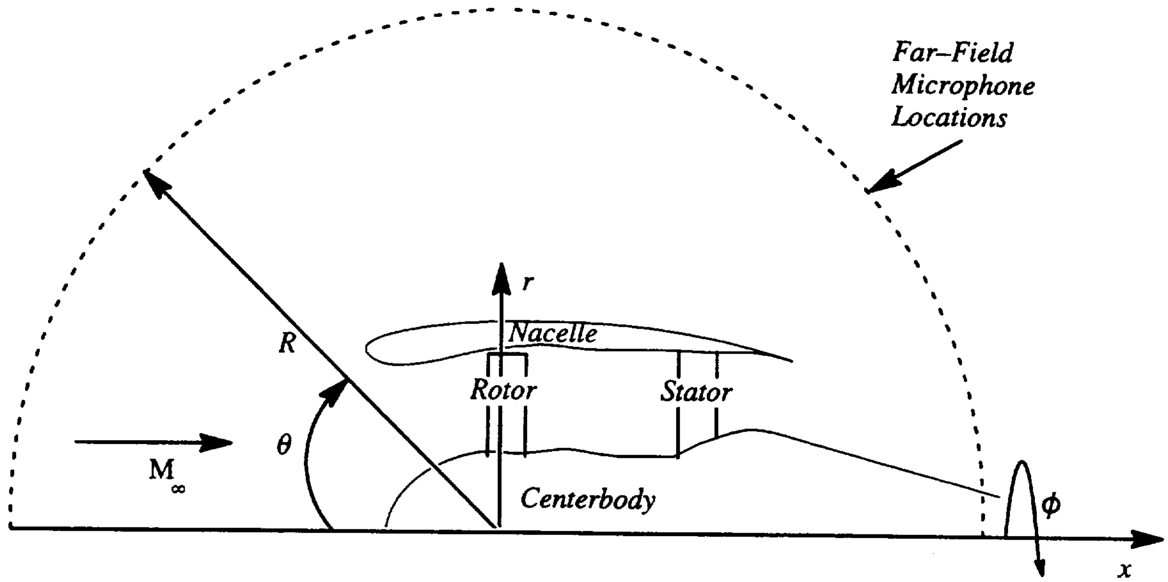

Figure 5: CUP3D Coordinate Systems

Thus, for the purposes of the present TFaNS system, consider a constant area annular duct with a uniform axial flow and swirl which takes the form of solid body swirl (i.e. $V_\theta = \Gamma r$). For this more limited case the wave types are:

$W = 1$ Upstream going pressure wave
$W = 2$ Downstream going pressure wave
$W = 3$ Vorticity wave 1 (downstream only) “no−w ” vorticity wave
$W = 4$ Vorticity wave 2 (downstream only) “no−u” vorticity wave

It should be noted that while the above more limited wave set is used by the present version of TFaNS, CUP3D can handle a more general set of modes by adjusting the number of wave types, so long as CUP3D uses the same set of wave types for all acoustic elements.

We will now discuss how the above more limited wave set is represented in the present version of TFaNS. The concepts presented below also apply to CUP3D with a more general set of modes.

3.2.1 State Vector Conventions

The acoustic pressure modes for coupling are of the general form:

$$\psi_{m\mu}(r)e^{i\left(m\phi + \hat{\gamma}_{nk\mu}x - nB\Omega t\right)} \tag{3}$$

Where:

$\psi_{m\mu}$ = *mode eigenfunction*
n = *harmonic of blade passing frequency*
m = *circumferential mode order* = $nB - kV$
B = *number of rotor blades*
V = *number of stator vanes*
μ = *radial mode order*
$\hat{\gamma}_{nk\mu}$ = *axial wavenumber normalized by the duct radius*
x = *x in Figure 5 but normalized by the duct radius.*

The full expression for perturbation pressure for a given wave type at any interface plane, P, (such as upstream going or downstream going waves) is;

$$p^P_W(x,r,\phi,t) = p_\infty \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} A^P_{Wnk\mu} \psi^P_{Wm\mu}(r) e^{i\left[m\phi + \tilde{\gamma}^P_{Wnk\mu}\left(x - x^P_W\right) - nB\Omega t\right]} \tag{4}$$

The "no−w" vorticity waves have no radial ("w") velocity component. The defining velocity component for this wave is in the axial direction and takes the form:

$$u^P_3(r,\phi,x,t) = c_\infty \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} A^P_{3nk\mu} U_{3\mu}(r) e^{i\left[m\phi + \tilde{\gamma}^P_{3nk\mu}\left(x - x^P_W\right) - nB\Omega t\right]} \tag{5}$$

The "no−u" vorticity waves (vorticity with no axial velocity component) are defined with the velocity component in the radial direction:

$$w^P_4(r,\phi,x,t) = c_\infty \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} A^P_{4nku} W_{4u}(r) e^{i\left[m\phi + \tilde{\gamma}^P_{4nk\mu}\left(x - x^P_W\right) - nB\Omega t\right]} \tag{6}$$

For coupling purposes these equations are assumed to be valid in the vicinity of each coupling plane, P. In these equations all lengths are normalized by the duct radius, r_d. The perturbation pressure mode amplitudes, $A^P_{Wnk\mu}$, are normalized by the far−field static pressure, p_∞ (W=1 or 2). The perturbation velocity for the vorticity modes (W= 3 or 4) are normalized by the far−field acoustic speed, c_∞. The acoustic pressure, p, has the same dimensions as p_∞. The velocities, u or w, have the dimensions as c_∞. The index n counts harmonics of blade passing frequency, m is the circumferential mode order, and μ is the radial mode index. The superscript, P, denotes the interface plane where this equation is being applied. The subscript, W, denotes the wave type.

An example of the waves associated with an interface plane is shown in Figure 6 for an isolated stator element. Consistent with Figure 1 where interface planes are numbered from inlet to aft, interface plane 2 represents the interface plane between the rotor and stator acoustic elements. The interface between the stator element and aft radiation element is plane 3. Note that the elements of the state vector, $A^P_{Wnk\mu}$, shown in this figure have a superscript which denotes the interface plane, P, and a subscript which denotes the wave type, W. The source vector elements, $\mathbf{B}$, are the mode amplitudes resulting from a prescribed input (such the response of a stator to an incoming wake).

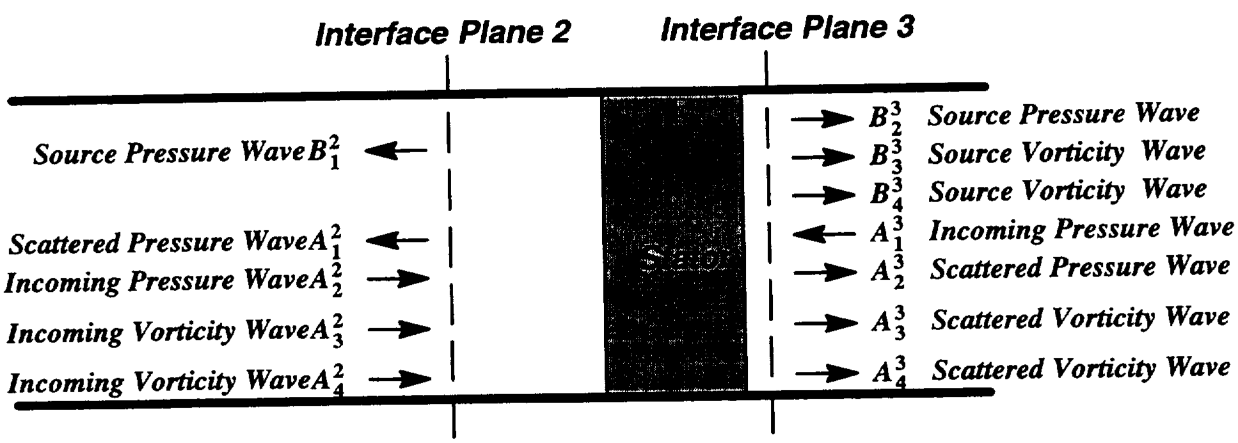

Figure 6: Mode Scattering through an Isolated Stator

To prevent the amplitudes of decaying waves from becoming too large, the axial origin of Equations (4) to (6), x_W^P, varies with interface plane and wave type. For a given acoustic element (e.g. Figure 6), the origin for each wave type is where each wave exits an acoustic element. For example, in Figure 6, the upstream pressure waves, A_1^2, have their origins at interface plane 2 since the upstream pressure wave emerges from the element at interface plane 2. Note that A_1^2, for example, is a compressed notation collecting all of the state vector elements, $A_{1nk\mu}^2$.

Since Equation (4) is the modal representations of acoustic pressure in an annular duct, a form for the mode eigenfunction, ψ, must be specified. Reference 11 uses the form below for the mode eigenfunctions for an annular duct with a mean axial flow. These mode eigenfunctions were originally developed by Tyler and Sofrin (Reference 13) and take the form:

$$\psi_{m\mu}(r) = C_{m\mu} J_{m\mu}\left(\kappa_{m\mu} r\right) + D_{m\mu} Y_{m\mu}\left(\kappa_{m\mu} r\right) \tag{7}$$

where $C_{m\mu}$ and $D_{m\mu}$ are constants determined by boundary conditions at the duct walls. For this case $\kappa_{m\mu}$ is the mode eigenvalue and the mode eigenfunction is $\psi_{m\mu}$ where these modes are normalized so that the maximum value of the mode eigenfunction is always a positive one (i.e. $\psi_{max} = +1$). The axial wavenumber non-dimensionalized by r_d, $\tilde{\gamma}^P_{W_{nk\mu}}$, is therefore found to be:

$$\tilde{\gamma}^P_{1,2_{nk\mu}} = \frac{-M_x(nBM_t) \mp \sqrt{(nBM_t)^2 - \left(1 - M_x^2\right)\left\{\kappa_{m\mu}\right\}^2}}{1 - M_x^2} \tag{8}$$

where upstream pressure waves ($W = 1$) use the upper sign and downstream pressure waves ($W = 2$) use the lower sign. If Equations (7) and (8) are modified with a kinematic transformation to include solid body swirl (as in Reference 2), then Equation (7) does not change. Only Equation (8) changes to become:

$$\tilde{\gamma}^P_{1,2_{nk\mu}} = \frac{-M_x(nBM_t - mM_s) \mp \sqrt{(nBM_t - mM_s)^2 - \left(1 - M_x^2\right)\left\{\kappa_{m\mu}\right\}^2}}{1 - M_x^2} \tag{9}$$

where M_s = Swirl Mach Number at the fan tip.

In the present version of TFaNS (see Reference 2), the "no–w" vorticity wave radial mode function, U_3, from Equation (5) is formulated as a series of cosine waves:

$$U_{3_\mu}(r) = \cos\left(\frac{\mu\pi\left(r - r_h\right)}{r_d - r_h}\right) \tag{10}$$

To prevent this mode function from becoming infinite when $m = 0$ in TFaNS, the radial mode function in TFaNS uses the form:

$$mU_{3_\mu} = m\cos\left(\frac{\mu\pi\left(r - r_h\right)}{r_d - r_h}\right) \tag{11}$$

Likewise the "no–u" vorticity wave radial mode function takes on the form:

$$mW_{4_\mu}(r) = \frac{m}{r/r_d}\cos\left(\frac{\mu\pi\left(r - r_h\right)}{r_d - r_h}\right) \tag{12}$$

The axial wave numbers (non− dimensionalized by r_d) associated these waves are:

$$\tilde{\gamma}^{P}_{3_{nku}} = \tilde{\gamma}^{P}_{4_{nku}} = \frac{nBM_t - mM_s}{M_x} \tag{13}$$

which corresponds to pure convection of the waves.

The mode eigenfunctions presented in Equations (7), (9), (11), and (13) are presently used by the SOURCE3D source code (Reference 2) as part of the TFaNS Tone FAn Noise design/prediction System. The radiation codes utilize Equations (7) and (8) since flow in the inlet and nozzle is swirl free. Kousen (Reference 14) developed modes which include arbitrary shear and swirl flows. In the future, these modes, or modes like them, will be utilized in a version of the CUP3D code.

CUP3D assumes that all pressure mode eigenfunctions, no matter how they are formulated, are consistent with Equation (4). CUP3D also assumes that all vorticity modes are consistent with Equations (5) and (6). In addition all pressure and vorticity modes are normalized so that the maximum value of the mode eigenfunction is always a positive one (i.e. $\psi_{max} = +1$). Thus, CUP3D is presently capable of accepting other modes (e.g. Reference 14) assuming that the number of wave types, W, is finite and a counting system for the wave types can be consistent for all acoustic property files entering CUP3D.

3.2.2 Scattering Coefficient Conventions

The explanation of the "Kinematics of Modal Scattering" in Reference 1 discusses how modes may be scattered into other wave types. Briefly in the CUP3D code:

Inlets and Nozzles:	$n, k, \mu \rightarrow n, k, \mu'$
Stators	$n, k, \mu \rightarrow n, k', \mu'$
Rotors	$n, k, \mu \rightarrow n', k, \mu'$

The arrow means "is scattered into" so that for example for a stator, an (n, k, μ) mode is scattered into a (n, k', μ') mode. Therefore stators scatter on circumferential order via k and radial mode order, μ. However, they do not scatter on frequency so that for stators n does not scatter into n'. However, rotors are seen to scatter on harmonic via n and radial mode order, μ, while not scattering on k.

Scattering Coefficients, S, represent the transmission and reflection of waves by an acoustic element. An example of mode scattering is shown in Figure 6 for an isolated stator element. This figure was discussed in detail in the previous section. Waves entering the stator acoustic element in this example are scattered into other waves by the element and then output. For example, the A_2^2 wave can be scattered into the A_1^2 wave per the notation in Reference 1 by:

$$A^2_{1nk'\mu'} \leftarrow S^{22}_{12nk'\mu';nk\mu} A^2_{2nk\mu} \tag{14}$$

This is the expression for one of the modes (n, k, μ) being scattered by this acoustic element into the mode (n, k', μ'). All other modes are scattered as well within the rules for wave scattering given above earlier in this section.

Note that when mode scattering occurs on harmonic (for a rotor element), noise is scattered into both positive and negative harmonics (i.e. $-\infty < n' < \infty$). Thus a positive harmonic, n, may scatter into a negative harmonic, n'. This must be taken into account in the scattering matrix output.

For the purpose of completeness, the total set of equations for a coupled system of inlet radiation, rotor, stator, and aft radiation is given by the matrix equation below. This is the same as Figure 4 in Reference 1 except for an additional vorticity wave type. Dots indicate blocks of zeros.

$$
\begin{array}{c}
\text{Interface Plane 1} \\[6em] \text{Interface Plane 2} \\[6em] \text{Interface Plane 3}
\end{array}
\begin{bmatrix} A_1^1 \\ A_2^1 \\ A_3^1 \\ A_4^1 \\ - \\ A_1^2 \\ A_2^2 \\ A_3^2 \\ A_4^2 \\ - \\ A_1^3 \\ A_2^3 \\ A_3^3 \\ A_4^3 \end{bmatrix}
=
\begin{array}{c}
\overbrace{\text{Inlet Radiation}} \quad \overbrace{\qquad \text{Blade Row} \qquad} \quad \overbrace{\qquad \text{Vane Row} \qquad} \quad \overbrace{\text{Aft Radiation}} \\
\left[\begin{array}{cccc|cccc|cccc}
\cdot & S_{12}^{11} & S_{13}^{12} & S_{14}^{12} & S_{11}^{12} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
S_{21}^{11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
S_{31}^{11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
S_{41}^{11} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\hline
\cdot & \cdot & \cdot & \cdot & \cdot & S_{12}^{22} & S_{13}^{22} & S_{14}^{22} & S_{11}^{23} & \cdot & \cdot & \cdot \\
\cdot & S_{22}^{21} & S_{23}^{21} & S_{24}^{21} & S_{21}^{22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & S_{32}^{21} & S_{33}^{21} & S_{34}^{21} & S_{31}^{22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & S_{42}^{21} & S_{43}^{21} & S_{44}^{21} & S_{41}^{22} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\hline
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & S_{12}^{33} & S_{13}^{33} & S_{14}^{33} \\
\cdot & \cdot & \cdot & \cdot & \cdot & S_{22}^{32} & S_{23}^{32} & S_{24}^{32} & S_{21}^{33} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & S_{32}^{32} & S_{33}^{32} & S_{34}^{32} & S_{31}^{33} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & S_{42}^{32} & S_{43}^{32} & S_{44}^{32} & S_{41}^{33} & \cdot & \cdot & \cdot
\end{array}\right]
\end{array}
\times
\begin{bmatrix} A_1^1 \\ A_2^1 \\ A_3^1 \\ A_4^1 \\ - \\ A_1^2 \\ A_2^2 \\ A_3^2 \\ A_4^2 \\ - \\ A_1^3 \\ A_2^3 \\ A_3^3 \\ A_4^3 \end{bmatrix}
+
\begin{bmatrix} B_1^1 \\ B_2^1 \\ B_3^1 \\ B_4^1 \\ - \\ B_1^2 \\ B_2^2 \\ B_3^2 \\ B_4^2 \\ - \\ B_1^3 \\ B_2^3 \\ B_3^3 \\ B_4^3 \end{bmatrix}
\qquad (15)
$$

Each S value includes all harmonics (both positive and negative), and circumferential and radial mode orders.

As discussed earlier, this equation can also be represented in a condensed form by Equation 15 in Reference 1:

$$\boldsymbol{A} = \boldsymbol{S}\boldsymbol{A} + \boldsymbol{B} \qquad (16)$$

Where:

$\boldsymbol{A}$ = *State Vector Array*
$\boldsymbol{S}$ = *Scattering Matrix*
$\boldsymbol{B}$ = *Source Vector Array*

Equations (4) to (6) suggest that all modes and harmonics are being considered in the calculation. However, solution of Equation (15), requires the truncation of the series for a finite size matrix. This is partly justified by the fact that acoustic modes are "cut off" above a certain mode order. For the acoustic modes given by Equation (7) and Equation (8) cutoff occurs when the the term under the square root sign in Equation (8) becomes negative, i.e.

$$(nBM_t)^2 - \left(1 - M_x^2\right)\left\{\kappa_{m\mu}\right\}^2 < 0 \qquad (17)$$

Another way of expressing "cut off" is via the cutoff ratio, ξ, which indicates cut off when $\xi < 1$. For modes in a constant area annular duct with a mean axial flow:

$$\xi = \left| \frac{nBM_t}{\kappa_{m\mu}\sqrt{1 - M_x^2}} \right| \tag{18}$$

Likewise for Equation (9):

$$\xi = \left| \frac{nBM_t - mM_s}{\kappa_{m\mu}\sqrt{1 - M_x^2}} \right| \tag{19}$$

For the purposes of CUP3D, all the cuton modes up to some sound harmonic, n, should be included in the Acoustic Properties File. In addition, a certain number of cutoff modes should be included. The issue of how many modes are actually required, however, will be the subject of further study. Therefore Equation (4) becomes:

$$p_W^P(x,r,\phi,t) = p_\infty \sum_{n=-NHT}^{NHT} \sum_{k=KMIN}^{KMAX} \sum_{\mu=0}^{MUMAX} A_{Wnk\mu}^P \psi_{Wm\mu}^P(r) e^{i\left[m\phi + \hat{\gamma}_{Wnk\mu}^P\left(x - x_W^P\right) - nB\Omega t\right]} \tag{20}$$

Where:

NHT = *number of harmonics being run.*
KMIN = *minimum value of k being run (different for each harmonic)*
KMAX = *maximum value of k being run (different for each harmonic)*
MUMAX = *maximum value of μ (different for each harmonic and k−value).*

For vorticity waves Equation (13) indicates that there is no cutoff. This creates a question of how many vorticity waves are required for the solution. In the present system, the, (n,k,μ), modes are used for pressure and vorticity waves. However, this issue will need to be investigated in the future. Also in the present SOURCE3D code (Reference 2) only the "no−u" vorticity wave are defined. Thus the Equation (5) becomes:

$$u_3^P(r,\phi,x,t) = c_\infty \sum_{n=-NHT}^{NHT} \sum_{k=KMIN}^{KMAX} \sum_{\mu=0}^{MUMAX} A_{3nku}^P U_{3\mu}(r) e^{i\left[m\phi + \hat{\gamma}_{3nku}^P\left(x - x_3^P\right) - nB\Omega t\right]} \tag{21}$$

It should be noted, these Fourier series includes both positive and negative harmonics. For cases where tones are not being scattered on harmonic (i.e. for stators and inlet and aft radiation), only the positive harmonics need to be used. This is because the negative harmonics are the complex conjugates of the positive harmonics. However, for rotor scattering, n' must include both the positive and negative harmonics. Thus the scattering coefficients for the various types of elements take on the form:

Inlet Radiation: n > 0, n' > 0: $A_{W'nk\mu'}^{P'} \leftarrow S_{W'Wnk\mu';nk\mu}^{P'P} A_{Wnk\mu}^P$ (22)

Rotor: n > 0, −NHT< n' < NHT: $A_{W'n'k\mu'}^{P'} \leftarrow S_{W'Wn'k\mu';nk\mu}^{P'P} A_{Wnk\mu}^P$ (23)

Stator: n > 0, n' > 0: $A_{W'nk'\mu'}^{P'} \leftarrow S_{W'Wnk'\mu';nk\mu}^{P'P} A_{Wnk\mu}^P$ (24)

Aft Radiation: n > 0, n' > 0: $A_{W'nk\mu'}^{P'} \leftarrow S_{W'Wnk\mu';nk\mu}^{P'P} A_{Wnk\mu}^P$ (25)

These expressions should be used in formulating the scattering coefficients for CUP3D. Scattering coefficients are found by inputting a unit value for $A^{P}_{Wnk\mu}$ and determining how much is reflected and transmitted through the element including all pressure waves and vorticity waves. Thus by setting $A^{P}_{Wnk\mu} = 1.0 + i0.0$ and finding $A^{P'}_{W'n'k'\mu'}$, we can determine $S^{P'P}_{W'W_{n'k'\mu';nk\mu}}$ by using expressions (22) through (25).

The source vector elements, $B^{P}_{Wnk\mu}$, are the mode amplitudes resulting from a prescribed outside influence such as a rotor wake interacting with a stator vane. ***B*** is defined by equations (20) and (21) with $A^{P}_{Wnk\mu}$ replaced by $B^{P}_{Wnk\mu}$.

3.3 REMOVING NEGATIVE HARMONICS FROM CUP3D

The purpose of this section is to reduce the size of the scattering matrix, S, in Equations (15) and (16), by eliminating the need to use negative harmonics in the calculation. The end of the previous section pointed out the need to include both positive and negative harmonics in Equation (20). However, for fan tones, the negative harmonics are complex conjugates of the positive harmonics. This is the basis for the removal of negative harmonics from the calculation. One exception to this as pointed out in the previous section, is that the rotor can scatter modes with positive harmonics into modes with negative harmonics. This is shown in Equation (23) where $S^{P'P}_{W'Wn'k\mu';nk\mu}$ can have the n'th harmonic scatter into the n'th negative harmonic. This aspect of the problem will be included in the discussion below.

We begin with Equation (15) and rewrite it so as to separate the positive harmonics (denoted by a "+" superscript) from the negative harmonics (denoted by a "−" superscript) so that:

$$\begin{bmatrix} A^- \\ A^+ \end{bmatrix} = \begin{bmatrix} S^{--} & S^{-+} \\ S^{+-} & S^{++} \end{bmatrix} \begin{bmatrix} A^- \\ A^+ \end{bmatrix} + \begin{bmatrix} B^- \\ B^+ \end{bmatrix} \tag{26}$$

Note that in this equation a scattering coefficient such as S^{-+} denotes a positive harmonic, n, scattering into a negative harmonic, n', as is found in a rotor.

It can be shown that first line in Equation (26) is the complex conjugate of the second line in Equation (26). Consequently only one of these lines is needed to calculate a solution. Thus we can use the second line:

$$A^+ = S^{+-}A^- + S^{++}A^+ + B^+ \tag{27}$$

Note that all the terms in this equation include only positive harmonic except the term $S^{+-}A^-$. Also note that:

$$S^{+-} = \{S^{-+}\}^* \text{ and } A^- = \{A^+\}^* \tag{28}$$

so that

$$A^+ = \{S^{-+}\}^* \{A^+\}^* + S^{++}A^+ + B^+ \tag{29}$$

Expanding Equation (29) into real and imaginary parts and separating the real parts and imaginary parts of the equation, we get:

$$\begin{bmatrix} A_R^+ \\ A_I^+ \end{bmatrix} = \begin{bmatrix} S_R^{++} + S_R^{-+} & -S_I^{++} - S_I^{-+} \\ S_I^{++} - S_I^{-+} & S_R^{++} - S_R^{-+} \end{bmatrix} \begin{bmatrix} A_R^+ \\ A_I^+ \end{bmatrix} + \begin{bmatrix} B_R^+ \\ B_I^+ \end{bmatrix} \tag{30}$$

The R subscript denotes the real part of the variable whereas the subscript, I, denotes the imaginary part. Note that in this equation only positive harmonics of BPF are needed as input waves. Also only a rotor will create S^{-+} coefficients because only a rotor scatters on harmonic. CUP3D uses Equation (30) in its calculation of the state vector, A.

3.4 CALCULATION OF POWER LEVELS & FAR–FIELD DIRECTIVITIES

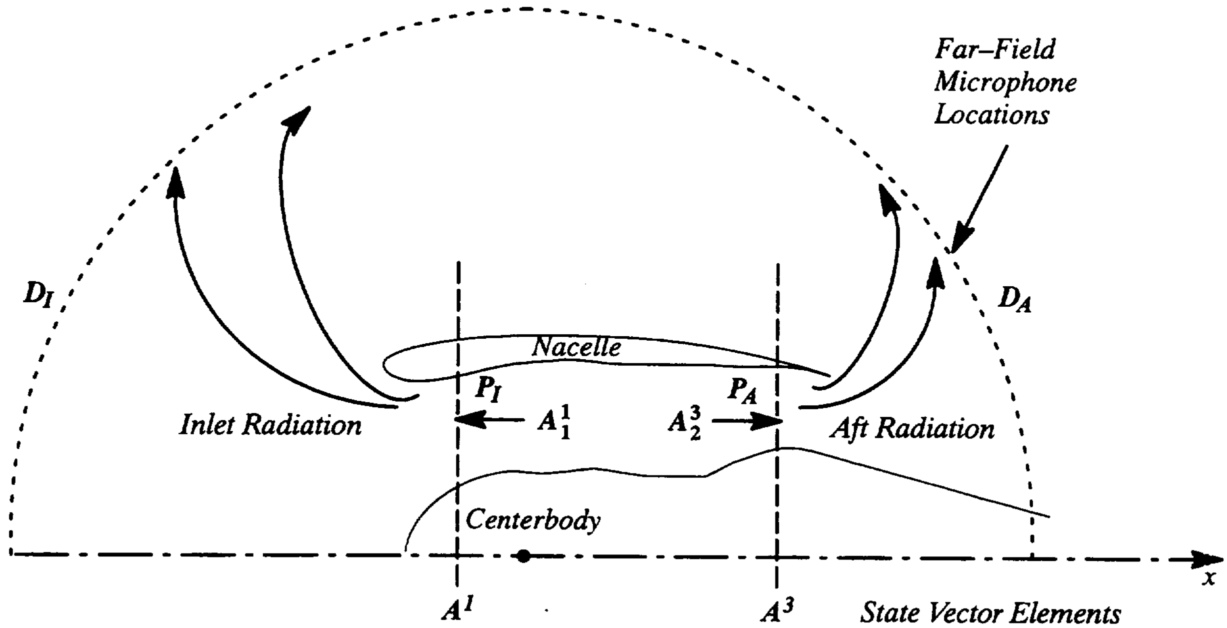

Figure 7: Calculation of Far–Field Noise From State Vector

Power levels and far–field directivities are computed from state vector elements at interface planes just upstream and just downstream of the noise source. To illustrate the application of this computation, the geometry of Figure 1 is utilized. However, CUP3D is not limited to this geometry. For the geometry of Figure 1, plane 1 is the upstream interface plane just in front of the fan and plane 3 is the downstream interface plane just aft of the FEGV. Figure 7 shows only the items directly related to the power level and far–field calculations. The state vector elements in Figure 7 are denoted by A_1^1 for the upstream pressure waves and A_2^3 for the downstream pressure waves.

Sound power can be calculated in concept using the equation form below:

$$\textit{Inlet Power Level}, \Pi_1^1 \sim |A_1^1|^2[P_I] \qquad \textit{Aft Power Level}, \Pi_2^3 \sim |A_2^3|^2[P_A] \tag{31}$$

where P_I and P_A denote inlet and aft power level multiplication factors, respectively.

Likewise, far–field pressure directivities can be calculated in concept by applying the equation form below:

$$\textit{Inlet Pressure Directivity}, P_n \sim [A_1^1][D_I] \qquad \textit{Aft Pressure Directivity}, P_n \sim [A_2^3][D_A] \tag{32}$$

where D_I and D_A are the inlet and aft far–field directivity shapes (based on unit input), respectively.

The sections which follow will discuss the detailed sound power level and far–field directivity equations. Derivations for sound power may be found in Appendix II and derivations for far–field pressure directivities may be found in Appendix IV.

3.4.1 Power Level Calculation

For purposes of the power level calculation only, the following assumptions are made in the vicinity of the interface planes where the calculation is taking place:

- Locally constant area annular duct,
- Locally uniform axial flow.

As a result, for the sound power calculation, the standard Tyler/Sofrin modes (Reference 13) are being used. A more complicated formulation may be developed in the future. Appendices II and III go into further detail on the equations below.

From Equation (31) and Appendix II, upstream and downstream sound power take on the form:

$$\hat{\Pi}^{P}_{Wnk\mu} = \left|A^{P}_{Wnk\mu}\right|^{2} P^{P}_{Wnk\mu} \tag{33}$$

where $\hat{\Pi}^{P}_{Wnk\mu}$ is the non–dimensional sound power for each acoustic mode and harmonic of blade passing frequency (BPF). This form of the power is non–dimensionalized by $p_{\infty}c_{\infty}(2r_d)^2$ or $p_{\infty}\sqrt{\gamma_a R_a T_{\infty}}\,(2r_d)^2$. $A^{P}_{Wnk\mu}$ is a state vector element where, for the example in Figure 7, $P = 1$, $W = 1$ for upstream pressure waves, and $P = 3$, $W = 2$ for downstream pressure waves. The power level multiplication factor, $P^{P}_{Wnk\mu}$, is given by:

$$P^{P}_{Wnk\mu} = \frac{\pi p_{\infty}}{\gamma_a p_P}\sqrt{\frac{T_P}{T_{\infty}}}\left[\left(1 + M^{2}_{x_P}\right)\hat{R}^{P}_{Wnk\mu} + M_{x_P} + M_{x_P}\left|\hat{R}^{P}_{Wnk\mu}\right|^{2}\right]C_{m\mu} \tag{34}$$

where the axial Mach Number at plane P, M_{x_P}, is positive downstream and:

$$\hat{R}^{P}_{Wnk\mu} = \frac{\tilde{\gamma}^{P}_{Wnk\mu}}{\left(nBM_{t_P} - M_{x_P}\tilde{\gamma}^{P}_{Wnk\mu}\right)} \tag{35}$$

$\tilde{\gamma}^{P}_{Wnk\mu}$ is the axial wavenumber as given by Equation (8) but repeated here as:

$$\tilde{\gamma}^{P}_{Wnk\mu} = \frac{-M_{x_P}nBM_{t_P} \mp \sqrt{\left(nBM_{t_P}\right)^{2} - \left(1 - M^{2}_{x_P}\right)\left\{\kappa_{m\mu}\right\}^{2}}}{1 - M^{2}_{x_P}} \tag{36}$$

$C_{m\mu}$ is the radial integral which includes modes of the form given by Equation (7) such that:

$$\int_{\sigma}^{1} \psi^{2}_{m\mu} r\,dr = C_{m\mu} \tag{37}$$

where r is the radius non-dimensionalized by r_d. Using the assumptions above, $C_{m\mu}$ can be integrated analytically (see Equation (III.13) in Appendix III) as:

$$C_{m\mu} = \frac{1}{2}\left[\left(1 - \frac{m^2}{\kappa_{m\mu}^2}\right)\psi_{m\mu}^2(\kappa_{m\mu}) - \left(\sigma^2 - \frac{m^2}{\kappa_{m\mu}^2}\right)\psi_{m\mu}^2(\kappa_{m\mu}\sigma)\right] \tag{38}$$

where $\psi_{m\mu}(\kappa_{m\mu})$ and $\psi_{m\mu}(\kappa_{m\mu}\sigma)$ are the tip and hub mode eigenfunction values respectively. Note that each mode eigenfunction, ψ, is normalized such that $\psi_{\max} = +1$.

The total harmonic sound power level is the sum of the modal powers:

$$\hat{\Pi}^P_{W_n} = \sum_{k=-\infty}^{+\infty} \sum_{\mu=0}^{\infty} \left|A^P_{Wnk\mu}\right|^2 P^P_{Wnk\mu} \tag{39}$$

To redimensionalize this non-dimensional power multiply by: $p_\infty \sqrt{\gamma_a R_a T_\infty}\,(2r_d)^2$. The units in the CUP3D code are: $(lb_f/in^2)(ft/sec)(in^2) = ft-lb_f/sec$. To convert to Watts, multiply by 1.3558. Then to calculate sound power level, reference this power to 10^{-12} Watts. This is shown as:

$$Sound\ Power = 10\log\left[\frac{\hat{\Pi}^P_{W_n}\left(p_\infty \sqrt{\gamma_a R_a T_\infty}\,(2r_d)^2 \frac{ft-lb_f}{sec}\right)\left(1.3558\frac{Watts}{ft-lb_f/sec}\right)}{10^{-12}\,Watts}\right] \tag{40}$$

Where $R_a = (53.35\ ft\ lb_f/{}^0R\ lb_m)(32.174\ lb_m\ ft/sec^2\ lb_f) = 1716.5\ ft^2/sec^2\ {}^0R$ and $\gamma_a = 1.4$. Note that p_∞ is in lb_f/in^2, T_∞ is in ${}^0R$ and r_d is in inches.

3.4.2 Far-Field Directivity Calculation

The purpose of this section is to present the equations used by CUP3D to calculate far-field directivities using the state vector solution (i.e. mode amplitudes) from CUP3D. A derivation of these equations may be found in Appendix IV. Appendix V documents the changes which were needed to the Eversman inlet and aft radiation codes to make them compatible with CUP3D. Appendix V provides an example of how the radiation equations in this section are applied to actual radiation codes so they can interface with CUP3D.

The CUP3D code has been formulated to permit multiple upstream or downstream pressure wave types to exist in the duct. This is done in order to accommodate mode sets such as those in Reference 14. Modes in Reference 14 are formulated in radially dependent axial and swirl flows and are therefore more general than the modes we are currently using in TFaNS. The equations below for far-field directivities apply to these more general modes thus allowing them to be included in the far-field solution.

To calculate the far−field directivity two parameters are required:

1. The solution to the state vector, $A^{P}_{Wnk\mu}$, at the interface plane where a radiation acoustic element is connected to the noise source.

2. Directivity shapes for each propagating pressure wave type and mode from an inlet radiation calculation, $D_{IWnk\mu}(R,\theta)$ and an aft radiation calculation, $D_{AWnk\mu}(R,\theta)$. These are determined by inputting unit waves into a radiation code and then having the radiation code calculate far−field directivities.

The following far−field pressure directivities are calculated by CUP3D:

- Total far−field sound pressure level (SPL) directivity for each harmonic
- Inlet SPL directivity for each harmonic
- Aft SPL directivity for each harmonic
- Inlet, aft and total SPL directivities for each circumferential mode order at each harmonic (summed over all radial mode orders)
- Inlet, aft and total SPL directivities for each circumferential and radial mode order at each harmonic

To calculate far−field directivities, start with Equation (IV.24) from Appendix IV, which calculates the total far−field pressure directivity for each BPF harmonic:

$$P_n(R,\theta) = \sqrt{\sum_{k=-\infty}^{\infty} \left| \sum_{\mu=0}^{\infty} \sqrt{2}\, p_{Ink\mu} + \sum_{\mu=1}^{\infty} \sqrt{2}\, p_{Ank\mu} \right|^2} \tag{41}$$

This equation gives the root mean square of the total harmonic pressure pattern averaged circumferentially. Circumferential averaging is required so that the azimuthal location of noise measurement relative to a circumferential origin on the engine is not needed to perform the calculation. Multiple circumferential mode orders will cause azimuthal variations in the far−field directivity which are averaged by this equation.

To calculate the far−field SPL's, note that $P_n(R,\theta)$ are normalized by the far−field static pressure, p_∞. Therefore:

$$SPL_{n_{total}} = 20 \log\left(\frac{p_\infty P_n(R,\theta)}{P_{ref}} \right) \tag{42}$$

where $P_{ref} = 20\mu\text{Pa}$.

In Equation (41), $p_{Ink\mu}(R,\theta)$ refers to the inlet complex pressure directivity for each *(m,μ)* mode at the interface plane, *P*, where an inlet is connected to the noise source (e.g. in Figure 7, $P = 1$). This is calculated as:

$$p_{Ink\mu}(R,\theta) = \sum_{W=1}^{N_{UPSTR}} A^{P}_{Wnk\mu} D_{IWnk\mu}(R,\theta) \tag{43}$$

Where:
$A^{P}_{Wnk\mu}$ = *Upstream propagating pressure wave amplitudes from the state vector at interface plane, P*
$D_{IWnk\mu}(R,\theta)$ = *Mode inlet directivity shape functions*

CUP3D is configured to use multiple upstream pressure wave types so that these pressures are added together to obtain the complex pressure directivity for each *(m, μ)* mode, $p_{Ink\mu}(R,\theta)$. The number of forward propagating pressure wave types is given as N_{UPSTR}. Note that for all modal representations developed to date (e.g. Reference 14) only one type of forward propagating mode has been found (i.e. $N_{UPSTR} = 1$). However, CUP3D uses Equation (43) as shown to permit more upstream pressure wave types to be added, if necessary.

In Equation (41), $p_{Ank\mu}(R,\theta)$ refers to the aft complex pressure directivity for each *(m,μ)* mode at the interface plane, *P*, where the nozzle is connected to the noise source (e.g. in Figure 7, $P = 3$). This is calculated as:

$$p_{Ank\mu}(R,\theta) = \sum_{W=N_{UPSTR}+1}^{N_{PRESDIR}} A^{P}_{Wnk\mu} D_{AWnk\mu}(R,\theta) \tag{44}$$

Where:
$A^{P}_{Wnk\mu}$ = *Downstream propagating pressure wave amplitudes from the state vector at interface plane, P*
$D_{AWnk\mu}(R,\theta)$ = *Mode aft directivity shape functions*

CUP3D is configured to use multiple downstream pressure wave types so that these pressures are added together to obtain the complex pressure directivity for each *(m,μ)* mode, $p_{Ank\mu}(R,\theta)$. The number of propagating pressure wave types is assumed below to be $N_{PRESDIR}$. For the modes presently in TFaNS and discussed in Section 3.2, $N_{PRESDIR} = 2$.

Using Equation (41), the other types of directivities calculated by CUP3D can be formulated. To calculate the total inlet directivity (circumferentially averaged), set $p_{Ank\mu} = 0$ in Equation (41). This yields the summation over all inlet modes to get the inlet pressure directivity where:

$$P_{n_{inlet}}(R,\theta) = \sqrt{\sum_{k=-\infty}^{\infty} \left| \sum_{\mu=0}^{\infty} \sqrt{2}\, p_{Ink\mu} \right|^2} \tag{45}$$

The same can be done with aft directivity where:

$$P_{n_{aft}}(R,\theta) = \sqrt{\sum_{k=-\infty}^{\infty} \left| \sum_{\mu=0}^{\infty} \sqrt{2} p_{Ank\mu} \right|^2} \tag{46}$$

To determine total, inlet, and aft pressure directivities for each circumferential mode, Equations (41), (45), and (46), respectively, may be applied without the summation over the circumferential mode orders to get the root mean square pressure directivity for each circumferential mode order. Thus total, inlet, and aft circumferential mode pressure directivities become:

$$P_{nk}(R,\theta) = \sqrt{\left| \sum_{\mu=0}^{\infty} \sqrt{2} p_{Ink\mu} + \sum_{\mu=0}^{\infty} \sqrt{2} p_{Ank\mu} \right|^2} \tag{47}$$

$$P_{nk_{inlet}}(R,\theta) = \sqrt{\left| \sum_{\mu=0}^{\infty} \sqrt{2} p_{Ink\mu} \right|^2} \tag{48}$$

$$P_{nk_{aft}}(R,\theta) = \sqrt{\left| \sum_{\mu=0}^{\infty} \sqrt{2} p_{Ank\mu} \right|^2} \tag{49}$$

Note that these results are not circumferentially averaged since there is no circumferential variation of a single circumferential mode order.

The same process may be used to get total, inlet, and aft pressure directivities for particular *(m,μ)* modes Where equations (47), (48), and (49) become respectively:

$$P_{nk\mu}(R,\theta) = \sqrt{\left| \sqrt{2} p_{Ink\mu} + \sqrt{2} p_{Ank\mu} \right|^2} \tag{50}$$

$$P_{nk\mu_{inlet}}(R,\theta) = \sqrt{\left| \sqrt{2} p_{Ink\mu} \right|^2} \tag{51}$$

$$P_{nk\mu_{aft}}(R,\theta) = \sqrt{\left| \sqrt{2} p_{Ank\mu} \right|^2} \tag{52}$$

SPL's can then be calculated for these cases by inserting these pressure directivities into Equation (42).

CUP3D uses Equations (41) to (52) to calculate the various far–field directivities.

4. CONCLUDING REMARKS

A Tone Fan Noise Design/Prediction System (TFaNS) was developed. The purpose of this system is to predict tone noise emanating from a fan stage including the effects of reflection and transmission by the rotor and stator and by the duct inlet and nozzle. These effects have been added to an existing annular duct/isolated stator noise prediction capability.

TFaNS consists of:

- The codes that compute the acoustic properties (reflection and transmission coefficients) of the various "acoustic elements" (e.g. inlet, rotor, stator, nozzle). These properties are written to Acoustic Properties Files,
- CUP3D: Fan Noise Coupling Code that reads these files, solves the coupling problem, and outputs the desired noise predictions,
- AWAKEN: CFD/Measured Wake Postprocessor which reformats CFD wake predictions and/or measured wake data so they can be used by the system.

This report discussed the organization of TFaNS and then provided the technical documentation for the CUP3D Fan Noise Coupling Code. It also documented for code developers the file format for Acoustic Properties Files required as input to CUP3D. The Eversman radiation codes (which are presently part of TFaNS) were made available as an example to show how information for the Acoustic Properties Files could be created. This last portion of the report also acted to document the changes to the Eversman codes in order to make them part of TFaNS.

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LIST OF SYMBOLS

PARAMETERS

A = *State Vector*
$A^P_{Wnk\mu}$ = *State Vector Element*
B = *Source Vector*
$B^P_{Wnk\mu}$ = *Source Vector Element*
B = *number of blades*
c = *acoustic speed*
$D_{IWnk\mu}$ = *inlet directivity from an inlet radiation code at radius, R in the far−field given unit input.*
$D_{AWnk\mu}$ = *aft directivity from an aft radiation code at radius, R in the far−field given unit input.*
I = *instantaneous acoustic intensity*
$\bar{I}$ = *time average intensity of I*
J = *Bessel Function of the first kind*
k = *circumferential mode number from* $m = nB - kV$ *where* $-\infty < k < +\infty$
k_x = *axial wavenumber in the Eversman radiation codes*
$\hat{k}_x$ = *axial wavenumber non−dimensionalized by* r_d *in the Eversman radiation codes*
m = *circumferential mode order where m is positive in the direction of rotor rotation*
M_f = *mean axial Mach Number at the input plane (interface plane) of the Eversman radiation codes*
M_s = *tip swirl Mach number*
M_t = *tip rotational Mach number*
M_x = *mean axial Mach number*
n = *harmonic of blade passing frequency*
N_1 = *uncorrected fan speed (rpm)*
N_{1c} = *corrected fan speed (rpm), see Equation (V.5)*
N_h = *number of BPF harmonics being calculated (see Appendix II)*
N_p = *number of interface planes*
p = *acoustic pressure*
p_∞ = *reference pressure (static far−field pressure)*
p_P = *mean static pressure at interface plane, P*
$p^P_{Wnk\mu_{rad}}$ = *modal acoustic pressure in the Eversman radiation codes*
$p_{nk\mu}(R, \theta)$ = *far−field pressure directivity for a given mode and harmonic non−dimensionalized by* p_∞
$p_{Ink\mu}$ = *inlet far−field pressure directivity for a given mode and harmonic non−dimensionalized by* p_∞
$p_{Ank\mu}$ = *aft far−field pressure directivity for a given mode and harmonic non−dimensionalized by* p_∞
$P(R, \theta, \phi, t)$ = *time dependent pressure at a given point in the far−field non−dimensionalized by* p_∞
$P_n(R,\theta)$ = *Harmonic far−field pressure directivity non−dimensionalized by* p_∞
r = *radial direction non−dimensionalized by* r_d
$\bar{r}$ = *dimensional radius (inches)*
r_d = *Outer Duct Radius (inches)*
r_h = *Hub Duct Radius (inches)*
R = *Radial distance from the engine origin to some location in the far−field.*
R_a = *Ideal gas constant for air* = $(53.35\ ft\ lb_f/^0R\ lb_m)(32.174\ lb_m\ ft/sec^2\ lb_f) = 1716.5\ ft^2/sec^2\ ^0R$
$R^P_{Wnk\mu}$ = *relates acoustic velocity modes to acoustic pressure modes as in equation (II.5)*
$\hat{R}^P_{Wnk\mu}$ = *relates acoustic velocity modes to acoustic pressure modes (non−dimensional). see eqn. (II.6)*
S = *Scattering matrix*

$S^{P'P}_{W'Wn'k'\mu':nk\mu}$ = Scattering Coefficient where "primed" subscripts denote the wave which is scattered into.
t = time (sec)
T = mean static temperature (^0R)
T_{01} = total temperature in the far field or in the inlet (^0R)
u = acoustic velocity
u_3 = acoustic velocity in the axial direction
U = mean flow axial velocity
U_3 = Radial Mode Shape (mode eigenfunction) for the axial component of the acoustic velocity
V = number of vanes
V_θ = swirl flow velocity
w_4 = acoustic velocity in the radial direction
W_4 = Radial Mode Shape (mode eigenfunction) for the radial component of the acoustic velocity
x = axial direction (positive downstream) non−dimensionalized by r_d
X = axial direction (dimensional)
Y = Bessel Function of the second kind
γ = axial wavenumber
$\hat{\gamma}$ = axial wavenumber non−dimensionalized by r_d
γ_a = ratio of specific heats = 1.4 for air
Γ = constant used to calculate solid body swirl
η_r = Reduced frequency in the Eversman Radiation codes, see Equation (V.1)
θ = directivity angle of the far−field location where data is to be measured.
$\Theta_{nk}(\phi,t) = (nB - kV)\phi - nB\Omega t$ (See Appendix IV)
κ = unknown constant in Bessel's equation (Appendix III)
$\kappa_{m\mu}$ = Mode eigenvalue non−dimensionalized by r_d
μ = radial mode order
Π = acoustic sound power level
$\hat{\Pi}$ = acoustic sound power level non−dimensionalized by $\rho_\infty c_\infty (2r_d)^2$
ρ = mean flow static density
σ = $\frac{r_h}{r_d}$ = hub to tip ratio of the duct
ϕ = circumferential direction (positive in the direction of rotor rotation)
$\varphi^P_{Wnk\mu}$ = perturbation velocity potential mode amplitude in the Eversman radiation codes
$\tilde{\varphi}^P_{Wnk\mu}$ = perturbation velocity potential for a single mode in the Eversman radiation codes
$\overline{\varphi}^P_{Wnk\mu}(R,\theta)$ = far−field perturbation velocity potential mode directivity in the Eversman radiation codes
ψ = Radial Mode Shape (mode eigenfunction) for the pressure waves normalized by ψ_{max}
ψ_{max} = Maximum value of each radial mode shape (mode eigenfunction) which is defined as +1
ω = rotational frequency (rad/sec)
Ω = rotor rotational speed (rad/sec)

SUBSCRIPTS AND SUPERSCRIPTS

aft	= *Aft radiated direction*
f	= *At the input plane (interface plane) of the Eversman radiation codes*
inlet	= *Inlet radiated direction*
k	= *circumferential mode number from* $m = nB - kV$ *where* $-\infty < k < +\infty$
m	= *circumferential mode order where m is positive in the direction of rotor rotation*
n	= *harmonic number*
P	= *Interface Plane Number*
W	= *Wave type, e.g. for modes in a uniform axial flow with solid body swirl:*
	= *1 (upstream pressure), 2 (downstream pressure), 3 & 4 (downstream vorticity)*
μ	= *radial mode order*
∞	= *reference value (i.e. in the far-field)*

APPENDIX I: CREATING A CUP3D ACOUSTIC PROPERTIES FILE

I.1 ACOUSTIC PROPERTIES FILE GENERAL INFORMATION

Acoustic Properties Files are output by rotor, stator, inlet radiation and aft radiation computer codes for input into CUP3D. An Acoustic Properties File is required for each reflecting acoustic element in the system. No Acoustic Properties File is needed for a non–reflecting element.

For a code to create an Acoustic Properties File, the following information should be placed in a file in the order shown below. Note that the code generating this file is designated by a specific type (ICODE). Based on the type of code, the information required by CUP3D will vary.

The parts of the Acoustic Properties File are:

- **Header Cards:** Contain information about the run including the code which created the file, the acoustic modes contained in the file, and certain geometry and performance of the acoustic element. This information is split into information which is common to all elements and information which is common to a specific type of element.
- **Scattering Coefficients:** These are the reflection and transmission coefficients of the acoustic element. Definitions for these coefficients are given in Sections 3.2 along with a method for creating them.
- **Source Vector:** The source vector is output if the acoustic element is influenced by an outside force such as a wake interacting with a vane or blade. In this case acoustic mode amplitudes and vorticity waves are created which form the source vector and are placed in the Acoustic Properties File. Definitions for these coefficients are given in Sections 3.2.
- **Far–field Directivities:** If the acoustic element radiates noise to the far–field, then far–field directivities are placed in the Acoustic Properties File. Pressure directivities result from unit input for each duct mode into a radiation code. These directivities are given at a constant radius, *R*, (See Figure 5) from a user specified origin.

Multiple cases can be placed in the same file. In this situation, the first case contains all of the cards given below as defined by the type of acoustic element being considered. All of the cases which follow will then omit Card 1. The cases are then stacked one below the other. See stator example 2 in Section I.3.7.

The following terminology is utilized by the system (see Figure 3 and Figure 4):

- An acoustic element (or region) refers to a part of the system with acoustic properties such as a rotor, stator, inlet radiation or aft radiation. An Acoustic Properties File must be created for any acoustic element with acoustic reflection properties. An Acoustic Properties File is not needed for a non–reflecting region.
- An interface plane is the plane where two acoustic elements meet.
- Scattering coefficients correspond to reflection and transmission coefficients for a particular region (see Section 3.2).
- The Source vector corresponds to the mode amplitudes resulting from a prescribed outside influence such as a rotor wake interacting with a stator vane.

Variables in this file are non–dimensionalized using the following conventions:

- *Lengths:* Non–dimensionalized by fan tip diameter at fan leading edge (inches), DFAN.
- *Temperature:* Non–dimensionalized by the far–field static temperature (^{0}R) (far from the nacelle), TINF.
- *Pressure:* Non–dimensionalized by the far–field static pressure (psia) (far from the nacelle), PINF.
- *Vorticity :* Non–dimensionalized by c_∞ where $c_\infty = \sqrt{\gamma_a R_a \{TINF\}}$

I.2 ACOUSTIC PROPERTIES FILE ASSUMPTIONS

- Waves are assumed to be given to the code in the following order: upstream propagating waves first, then downstream propagating waves. Pressure waves are input before vorticity waves. Vorticity waves only propagate in the downstream direction. NWAVETYPE denotes the total number of waves which can propagate. NUPSTRE = number of upstream propagating pressure waves. NPRESDIR = the total number of propagating pressure waves. In the present code:

  NWAVETYPE = 4, NUPSTRE = 1, NPRESDIR = 2.

- All source vector information in the Acoustic Properties Files should be expressed as peak complex modal amplitudes.
- The code assumes that all acoustic elements were created with the same number of blades (NBLADE), vanes (NVANE), and fan rotational tip speed (VTIPC). Hub to tip ratios at the interface planes should be the same for the two connecting acoustic elements for the modes to be passed correctly between regions. The code, however, does not prevent hub to tip ratios from differing from region to region.
- The code presently assumes that all acoustic elements are in a single duct (such as in Figure 1). The concept of a core engine duct and bypass duct is not implemented in the code.
- The code is written so as to assume that all cases are being run in a user defined order by CUP3D. The same order is assumed for all acoustic elements. Therefore, if multiple cases were run by a stator code for example using a different order than was used for a rotor code, then CUP3D will not run correctly.
- Far field angles are assumed to be stored in the Acoustic Properties Files with the smallest angle first and the largest angle last. Zero degrees is assumed to be in front of the inlet.
- It is assumed that the user has chosen an origin for the entire calculation and that the origin is consistent for all Acoustic Properties Files being input into CUP3D. Far–field directivities should utilize this origin to specify distance from the engine and angles around the engine.

I.3 ACOUSTIC PROPERTIES FILE FORMAT

The discussion of the Acoustic Properties File format will be as follows. First the header cards common to all cases will be discussed in Section I.3.1. After that all other cards will be discussed based on the type of computer code which is outputting this file (i.e. inlet radiation code in Section I.3.2, rotor code in Section I.3.3, stator code in Section I.3.4, aft radiation code in Section I.3.5). Details of the scattering coefficient and source vector formats is found in Section I.3.6. Examples of Acoustic Properties Files are found in Section I.3.7. Also the method used to modify the Eversman radiation codes is given in Section V.2, Appendix V. Appendix V may be helpful towards the understanding of the process for creating files.

The input data file structure has distinct blocks of data referred to as cards. Each card begins on a new line and the input data is either free format or formatted as given in the card.

Note that a $*$ denotes parameters which must have the same value from element to element for the CUP3D code to run correctly. It is imperative that these parameters be defined exactly as given in this document.

Except where noted, all input is free format.

For all complex numbers the following format statement is used:

999 FORMAT(10E13.6)

I.3.1 Header Cards for all Acoustic Properties Files

All Acoustic Properties Files contain the header cards below.

Card 1: (for multiple cases this card is output only once with the first case)

- ICODE: Computer Code which created this file

  Note: ICODE is designed so that the first number denotes the type of code which created this file. The second number refers to which code created this file. For example: ICODE = 11: The first 1 refers to inlet radiation codes, the second 1 refers to the Eversman inlet radiation code.

  =10 Other Inlet Radiation Code

  =11 Eversman Inlet Radiation Code

  =12 Caruthers Inlet Radiation Code

  =20 Other Aft Radiation Code

  =21 Eversman Aft Radiation Code

  =22 Caruthers Aft Radiation Code

  =30 Other Rotor Code

  =31 SOURCE3D Rotor Code

  =40 Other Stator Code

  =41 SOURCE3D Stator Code

- NCASE: Number of "cases" where a case is defined as all harmonics for a given condition or configuration. Acoustic Properties Files are read in the order ICASE = 1, NCASE (see Card 2).

Card 2: ICASE: Case Number: FORMAT(1X,'Case Number',I3)

Card 3: CODETITLE: Title Card – identifying the case. Use the title from the code which was run. FORMAT(A80).

Card 4:

- NHT: Number of blade passing frequency harmonics run.

∗ VTIPC: Corrected fan rotational tip speed (ft/sec).

where: $VTIPC = \frac{\pi}{720} N_{1c}(DFAN)$

DFAN = Fan tip diameter (inches) at the fan leading edge

N_{1c} = Corrected Fan Rotational Speed (rpm) $= \dfrac{N_1}{\sqrt{\frac{T_{o1}}{518.67}}}$,

N_1 = Fan rotational speed (rpm)

T_{o1} = reference temperature (^{0}R) used to correct the fan speed to a "standard day". For zero Mach number in the far−field, this temperature is the static far−field ambient temperature,TINF.

Card 5:

∗ NBLADE: Number of rotor blades

∗ NVANE: Number of stator vanes.

Card 6: (KMIN(N,IWAVE), KMAX(N,IWAVE), N=1,NHT), IWAVE = 1,NWAVETYPE

Where for the case which was run:

KMIN = Minimum circumferential mode number, K, run for each harmonic,

KMAX = Maximum circumferential mode number, K, run for each harmonic where:

m = n*NBLADE − K*NVANE

n = harmonic number

m = circumferential mode order.

Card 7: (((M(N,K,IWAVE), K=KMIN(N,IWAVE), KMAX(N,IWAVE)), N=1,NHT), IWAVE = 1,NWAVETYPE): Circumferential mode order, m, where m is positive in the direction of rotor rotation.

Card 8: (((MUMAX(N,K,IWAVE), K=KMIN(N,IWAVE), KMAX(N,IWAVE)), N=1,NHT), IWAVE = 1,NWAVETYPE): Largest Value for μ for each circumferential mode order and harmonic. μ refers to the radial mode order where $0 \le \mu \le$ MUMAX.

Card 9: WRITE(IUNIT,999) ((((GAMMA(N,K,MU,IDIR), MU=0,MUMAX(N,K,IDIR)), K=KMIN(N,IDIR), KMAX(N,IDIR)), N=1,NHT), IDIR = 1,NPRESDIR): Complex Axial wave numbers for acoustic pressures directed upstream (IDIR=1) or downstream (IDIR=2) normalized by the duct radius. This parameter is presently input as one set of values per region. It is anticipated that at some time in the future, separate upstream and downstream GAMMA's will be input for each interface plane.

I.3.2 Inlet Radiation Code: 10 ≤ ICODE < 20

I.3.2.1 Header Cards

Information in Cards 11 through 16 are only used to confirm for the user which Acoustic Properties File the user has chosen. These parameters are not used in code calculations. While parameters in Card 10 are not presently required for code calculations, this information expected to be required for code calculations in the future.

Card 10:

- SIGMAR: Hub to tip ratio of the duct at XINTER. This number should, strictly speaking, be the same for two codes meeting at a given interface plane.
- XINTER: Axial location of the interface plane relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- MINF: Far−field axial Mach number (far from the nacelle, positive downstream)
- MDUCT: Mass averaged axial Mach number at the interface plane (positive downstream)

Card 11: NLINO: Number of acoustically lined elements (outer wall). Set this value equal to zero if there is no outer wall liner.

If (NLINO .GT. 0) THEN

Card 12:

- MBEGO: Outer wall element number for start of liner.
- XBEGO: Axial location where outer liner begins relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- XENDO: Axial location where outer liner ends relative to the origin (non−dimensionalized by DFAN) as specified by the user.

Card 13: WRITE(IUNIT,999) ZIMPO(I),I=MBEGO,NLINO+MBEGO: Liner impedance (Resistance, Reactance) for each element on outer wall non−dimensionalized by local ρc (i.e. density x acoustic speed).

ENDIF

Card 14: NLINI: Number of acoustically lined elements (inner wall). Set this value equal to zero if there is no inner wall liner.

If (NLINI .GT. 0) THEN

Card 15:

- MBEGI: Inner wall element number for start of liner.
- XBEGI: Axial location where inner liner begins relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- XENDI: Axial location where inner liner ends relative to the origin (non−dimensionalized by DFAN) as specified by the user.

Card 16: WRITE(IUNIT,999) ZIMPI(I),I=MBEGI,NLINI+MBEGI: Liner impedance (Resistance, Reactance) for each element on the inner wall non–dimensionalized by the local ϱc (i.e. density x acoustic speed).

ENDIF

I.3.2.2 Scattering Coefficients (see Section I.3.6)

Card 17: Write: FORMAT(1X,'Scattering Matrix and Source Vector')

Card 18: Write: FORMAT(1X,'Scattering Matrix')

Card 19: Scattering coefficients are written as follows:

```
C
C  WRITE SCATTERING MATRIX INFORMATION, S21, AT THE INTERFACE
C  PLANE DOWNSTREAM OF THIS REGION.  THESE SCATTERING
C  COEFFICIENTS CORRESPOND TO UPSTREAM PROPAGATING WAVES
C   SCATTERED INTO DOWNSTREAM PROPAGATING WAVES.  NOTE THAT
C  S31 & S41 ARE NOT WRITTEN SINCE THEY ARE VORTICITY WAVE BASED.
C  THERE ARE NO VORTICITY WAVES EMANATING FROM THE INLET
C  RADIATION CODE.  THUS S31 AND S41 ARE ZERO.
C
      DO 10 IW = 1,NUPSTRE
        DO 20 IWP = NUPSTRE+1,NPRESDIR
C
         WRITE(IUNIT,999)(((( S^DD_W'W(N, K, MU, MUP),
   &          MUP  =        0,MUMAX(N,K,IWP)),          <-μ'
   &          MU =         0,MUMAX(N,K,IW)),            <-μ
   &          K   = KMIN(N,IW),KMAX(N,IW)),             <-k
   &          N   = 1,        NHT)                      <-n
C
 20      CONTINUE
 10    CONTINUE
```

I.3.2.3 Source Vector (see Section I.3.6)

Card 20: IFORCE = 0 No Source Vector is Input
= 1 Source Vector is input

IF IFORCE = 1:

Card 21: Write: FORMAT(1X,'Source Vector')

Card 22: Upstream propagating pressure mode amplitudes normalized by PINF

```
    DO 10 IW = 1,NUPSTRE
      READ(IUNIT,999) ((( B^U_W(MU,K,N),
   &   MU  = 0,          MUMAX(N,K,IW)),          <-μ
   &   K   = KMIN(N,IW),KMAX(N,IW)),             <-k
   &   N   = 1,         NHT)                      <-n
10  CONTINUE
```

Card 23: Downstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = NUPSTRE + 1,NPRESDIR
     READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),          ← μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                ← k
  &    N   = 1,            NHT)                     ← n
10  CONTINUE
```

Card 24: Downstream propagating vorticity waves normalized by c_∞.

```
   DO 10 IW = NPRESDIR + 1, NWAVETYPE
     READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),          ← μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                ← k
  &    N   = 1,            NHT)                     ← n
10  CONTINUE
```

ENDIF

I.3.2.4 Far–field Directivities

Card 25: Write: FORMAT(1X,'Farfield Directivities')

Card 26:

- NANGLE: Number of polar angles (θ in Figure 5) in the directivity pattern. The code will interpolate these angles to form an array every 1^0 from $0^0 \leq \text{ANGLE} \leq 180^0$. In the present code, any angle which is within 25^0 of the aft most computational boundary will be discarded. This is consistent with the baffle boundary condition which presently is found in the Eversman Inlet Radiation code.
- DARRAY: Radial distance (R in Figure 5) from the origin non–dimensionalized by DFAN (origin specified by the user) of the polar directivity array.

Card 27: Far–field polar angle in degrees (0^0 is in front of the inlet)

- WRITE(IUNIT,*)(ANGLE(IANGLE), IANGLE = 1, NANGLE)

Card 28: Write: FORMAT(1X,'COMPLEX PRESSURE DIRECTIVITIES')

Card 29: Complex pressure directivities (real and imaginary parts of the directivities for each mode):

```
      DO 10 IW = 1,NUPSTRE
   C
        WRITE(IUNIT,999) (((( DIRECTIN(N,K,MU,IANGLE),
          IANGLE = 1,            NANGLE),
          MU     = 0,            MUMAX(N,K,IW)),
          K      = KMIN(N,IW), KMAX(N,IW))
          N      = 1, NHT)
    10  CONTINUE
```

I.3.3 Rotor Code: 30 ≤ ICODE < 40

I.3.3.1 Header Cards

Information in Cards 12 through 16 are only used to confirm for the user which Acoustic Properties File the user has chosen. These parameters are not used in code calculations. Parameters in Cards 10 and 11 (with the exception of X1 and X2) are used to calculate power in the code. X1 and X2 are not presently used for calculations in the code, but are expected to be needed in the future.

Card 10: Geometry and performance at the interface plane upstream of the region

- SIGMAR1: Hub to tip ratio of the duct at X1. This number should, strictly speaking, be the same for two codes meeting at a given interface plane.
- X1: Axial location of the interface plane upstream of the acoustic element relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- DIA1: Outer duct diameter of the interface plane upstream of the acoustic element (non−dimensionalized by DFAN).
- MX1: Mass averaged axial Mach number at the interface plane upstream of the acoustic element.
- MTIP1: tip rotational Mach number at the interface plane upstream of the acoustic element.
- PS1: Mass averaged static pressure at the interface plane upstream of the acoustic element (non−dimensionalized by PINF).
- TS1: Mass averaged static temperature at the interface plane upstream of the acoustic element (non−dimensionalized by TINF).

Card 11: Geometry and performance at the interface plane downstream of the region

- SIGMAR2: Hub to tip ratio of the duct at X2. This number should, strictly speaking, be the same for two codes meeting at a given interface plane.
- X2: Axial location of the interface plane downstream of the acoustic element relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- DIA2: Outer duct diameter of the interface plane downstream of the acoustic element (non−dimensionalized by DFAN).
- MX2: Mass averaged axial Mach number at the interface plane downstream of the acoustic element.
- MTIP2: tip rotational Mach number at the interface plane downstream of the acoustic element.
- PS2: Mass averaged static pressure at the interface plane downstream of the acoustic element (non−dimensionalized by PINF).
- TS2: Mass averaged static temperature at the interface plane downstream of the acoustic element (non−dimensionalized by TINF).

Card 12: NDAT: Number of streamlines where geometry is specified

Card 13: DIA(I),I=1,NDAT: Streamline diameter (non−dimensionalized by DFAN)

Card 14: RSOLIDITY(I),I=1,NDAT: Rotor solidity (rotor chord/rotor pitch)

Card 15: RALPHCH(I), I=1,NDAT: Rotor stagger angle relative to the circumferential direction (degrees). This is normally considered to be a number between 0 and 110 degrees.

Card 16: SSOC(I), I=1,NDAT: Upstream stator trailing edge to rotor leading edge aerodynamic Spacing to upstream stator aerodynamic chord for each streamline (Set equal to zero if there is no upstream stator cascade).

I.3.3.2 Scattering Coefficients (see Section I.3.6)

Card 17: Write: FORMAT(1X,'Scattering Matrix and Source Vector')

Card 18: Write: FORMAT(1X,'Scattering Matrix')

Card 19: Scattering coefficients as follows

```
C
C    WRITE SCATTERING MATRIX INFORMATION S12, S13, S14, AT THE
C    INTERFACE PLANE UPSTREAM OF THIS REGION. THESE SCATTERING
C    COEFFICIENTS CORRESPOND TO DOWNSTREAM PROPAGATING WAVES
C    ENTERING THE UPSTREAM INTERFACE PLANE AND BEING REFLECTED
C    BY THE ELEMENT BACK OUT AS UPSTREAM WAVES.
C
      DO 30 IW = NUPSTRE+1,NWAVETYPE
       DO 40 IWP = 1, NUPSTRE
        WRITE(IUNIT,999)((((( S^UU_W'W(N, NP, K, MU, MUP),
     &     MU  = 0,         MUMAX(N,K,IW)),                  ← μ
     &     MUP = 0,         MUMAX(IABS(NP),K,IWP)),          ← μ'
     &     K   = KMIN(N,IW),KMAX(N,IW)),                     ← k
     &     N   = 1,         NHT),                            ← n
     &     NP  = -NHT,      NHT)                             ← n'
 40    CONTINUE
 30   CONTINUE
C
C    WRITE SCATTERING MATRIX INFORMATION, S11, AT THE INTERFACE
C    PLANE UPSTREAM OF THIS REGION. THESE SCATTERING COEFFICIENTS
C    CORRESPOND TO UPSTREAM PROPAGATING WAVES AT THE
C    DOWNSTREAM INTERFACE BEING TRANSMITTED THROUGH THE
C    ACOUSTIC ELEMENT TO THE UPSTREAM INTERFACE
C
      DO 50 IW = 1, NUPSTRE
       DO 60 IWP = 1, NUPSTRE
        WRITE(IUNIT,999)((((( S^UD_W'W(N, NP, K, MU, MUP),
     &     MU  = 0,         MUMAX(N,K,IW)),                  ← μ
     &     MUP = 0,         MUMAX(IABS(NP),K,IWP)),          ← μ'
     &     K   = KMIN(N,IW),KMAX(N,IW)),                     ← k
     &     N   = 1,         NHT),                            ← n
     &     NP  = -NHT,      NHT)                             ← n'
C
 60    CONTINUE
 50   CONTINUE
```

```
C
C      WRITE S22, S32, S42, S23, S33, S43, S24, S34, S44. THESE COEFFICIENTS
C      CORRESPOND TO DOWNSTREAM PROPAGATING WAVES INPUT INTO THE
C      REGION AT THE UPSTREAM INTERFACE PLANE WHICH ARE
C      TRANSMITTED THROUGH THE ELEMENT TO THE DOWNSTREAM
C      INTERFACE PLANE
C
      DO 50 IW = NUPSTRE+1,NWAVETYPE
       DO 60 IWP = NUPSTRE+1,NWAVETYPE
         WRITE(IUNIT,999)((((( S^DU_W'W(N, NP, K, MU, MUP),
     &     MU  = 0,           MUMAX(N,K,IW)),                       ← μ
     &     MUP = 0,           MUMAX(IABS(NP),K,IWP)),               ← μ'
     &     K   = KMIN(N,IW),KMAX(N,IW)),                            ← k
     &     N   = 1,           NHT),                                 ← n
     &     NP  = -NHT,     NHT)                                     ← n'
 60     CONTINUE
 50   CONTINUE
C
C      WRITE SCATTERING MATRIX INFORMATION, S21, S31, S41, AT THE
C      INTERFACE PLANE DOWNSTREAM OF THIS REGION. THESE SCATTERING
C      COEFFICIENTS CORRESPOND TO UPSTREAM PROPAGATING WAVES
C      REFLECTED BY THE ACOUSTIC ELEMENT AS DOWNSTREAM
C      PROPAGATING WAVES.
C
      DO 70 IW = 1,NUPSTRE
       DO 80 IWP = NUPSTRE+1,NWAVETYPE
         WRITE(IUNIT,999)((((( S^DD_W'W(N, NP, K, MU, MUP),
     &     MU  = 0,           MUMAX(N,K,IW)),                       ← μ
     &     MUP = 0,           MUMAX(IABS(NP),K,IWP)),               ← μ'
     &     K   = KMIN(N,IW),KMAX(N,IW)),                            ← k
     &     N   = 1,           NHT),                                 ← n
     &     NP  = -NHT,     NHT)                                     ← n'
 80     CONTINUE
 70   CONTINUE
```

I.3.3.3 Source Vector (see Section I.3.6)

Card 20: IFORCE = 0 No Source Vector is Input
= 1 Source Vector is input

IF IFORCE = 1:

Card 21: Write: FORMAT(1X,'Source Vector')

Card 22: Upstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = 1,NUPSTRE
    READ(IUNIT,999) ((( B_W^U(MU,K,N),
  &    MU  = 0,           MUMAX(N,K,IW)),          ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),               ←k
  &    N   = 1,           NHT)                     ←n
10   CONTINUE
```

Card 23: Downstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = NUPSTRE + 1,NPRESDIR
    READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,           MUMAX(N,K,IW)),          ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),               ←k
  &    N   = 1,           NHT)                     ←n
10   CONTINUE
```

Card 24: Downstream propagating vorticity waves normalized by c_∞.

```
   DO 10 IW = NPRESDIR + 1, NWAVETYPE
    READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,           MUMAX(N,K,IW)),          ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),               ←k
  &    N   = 1,           NHT)                     ←n
10   CONTINUE
```

ENDIF

I.3.4 Stator Code: $40 \leq ICODE < 50$

I.3.4.1 Header Cards

Information in Cards 12 through 16 are only used to confirm for the user which Acoustic Properties File the user has chosen. These parameters are not used in code calculations. Parameters in Cards 10 and 11 (with the exception of X1 and X2) are used to calculate power in the code. X1 and X2 are not presently used for calculations in the code, but are expected to be needed in the future.

Card 10: Geometry and performance at the interface plane upstream of the region

- SIGMAR1: Hub to tip ratio of the duct at X1. This number should, strictly speaking, be the same for two codes meeting at a given interface plane.
- X1: Axial Location of the interface plane upstream of the acoustic element relative to the origin (non–dimensionalized by DFAN) as specified by the user.
- DIA1: Outer duct diameter of the interface plane upstream of the acoustic element (non–dimensionalized by DFAN)
- MX1: Mass averaged axial Mach number at the interface plane upstream of the acoustic element.
- MTIP1: tip rotational Mach number at the interface plane upstream of the acoustic element.
- PS1: Mass averaged static pressure at the interface plane upstream of the acoustic element (non–dimensionalized by PINF).
- TS1: Mass averaged static temperature at the interface plane upstream of the acoustic element (non–dimensionalized by TINF).

Card 11: Geometry and performance at the interface plane downstream of the region

- SIGMAR2: Hub to tip ratio of the Duct at X2. This number should, strictly speaking, be the same for two codes meeting at a given interface plane.
- X2: Axial location of the interface plane downstream of the acoustic element relative to the origin (non–dimensionalized by DFAN) as specified by the user.
- DIA2: Outer duct diameter of the interface plane downstream of the acoustic element (non–dimensionalized by DFAN).
- MX2: Mass averaged axial Mach number at the interface plane downstream of the acoustic element.
- MTIP2: tip rotational Mach number at the interface plane downstream of the acoustic element.
- PS2: Mass averaged static pressure at the interface plane downstream of the acoustic element (non–dimensionalized by PINF).
- TS2: Mass averaged static temperature at the interface plane downstream of the acoustic element (non–dimensionalized by TINF).

Card 12: NDAT: Number of Streamlines where geometry is specified

Card 13: DIA(I),I=1,NDAT: Streamline diameter (non–dimensionalized by DFAN)

Card 14: SSOLIDITY(I),I=1,NDAT: Stator Solidity (stator chord/stator pitch)

Card 15: SALPHCH(I), I=1,NDAT: Stator Stagger Angle relative to the circumferential direction (degrees). This is normally considered to be a number between 0 and 110 degrees.

Card 16: SSOC(I), I=1,NDAT: Upstream rotor trailing edge to stator leading edge aerodynamic Spacing to upstream rotor aerodynamic chord for each streamline (Set equal to zero if there is no upstream rotor).

I.3.4.2 Scattering Coefficients (see Section I.3.6)

Card 17: Write: FORMAT(1X,'Scattering Matrix and Source Vector')

Card 18: Write: FORMAT(1X,'Scattering Matrix')

Card 19: Scattering coefficients as follows

```
C
C    WRITE SCATTERING MATRIX INFORMATION S12, S13, S14, AT THE
C    INTERFACE PLANE UPSTREAM OF THIS REGION.  THESE SCATTERING
C    COEFFICIENTS CORRESPOND TO DOWNSTREAM PROPAGATING WAVES
C    ENTERING THE UPSTREAM INTERFACE PLANE AND BEING REFLECTED
C    BY THE ELEMENT BACK OUT AS UPSTREAM WAVES.
C
     DO 70 IW = NUPSTRE+1,NWAVETYPE
      DO 80 IWP = 1, NUPSTRE
       WRITE(IUNIT,999)((((( S^UU_W'W(N, K, KP, MU, MUP),
   &     MU  = 0,             MUMAX(N,K,IW)),              ←μ
   &     MUP = 0,             MUMAX(N,KP,IWP)),            ←μ'
   &     K   = KMIN(N,IW), KMAX(N,IW)),                    ←k
   &     KP  = KMIN(N,IWP),KMAX(N,IWP)),                   ←k'
   &     N   = 1,             NHT)                         ←n
 80   CONTINUE
 70  CONTINUE
C
C    WRITE SCATTERING MATRIX INFORMATION, S11, AT THE INTERFACE
C    PLANE UPSTREAM OF THIS REGION.  THESE SCATTERING COEFFICIENTS
C    CORRESPOND TO UPSTREAM PROPAGATING WAVES AT THE
C    DOWNSTREAM INTERFACE BEING TRANSMITTED THROUGH THE
C    ACOUSTIC ELEMENT TO THE UPSTREAM INTERFACE
C
     DO 90 IW = 1, NUPSTRE
      DO 100 IWP = 1, NUPSTRE
       WRITE(IUNIT,999)((((( S^UD_W'W(N, K, KP, MU, MUP),
   &     MU  = 0,             MUMAX(N,K,IW)),              ←μ
   &     MUP = 0,             MUMAX(N,KP,IWP)),            ←μ'
   &     K   = KMIN(N,IW), KMAX(N,IW)),                    ←k
   &     KP  = KMIN(N,IWP),KMAX(N,IWP)),                   ←k'
   &     N   = 1,             NHT)                         ←n
 100  CONTINUE
 90  CONTINUE
```

```
C
C     WRITE S22, S32, S42, S23, S33, S43, S24, S34, S44.  THESE COEFFICIENTS
C     CORRESPOND TO DOWNSTREAM PROPAGATING WAVES INPUT INTO THE
C     REGION AT THE UPSTREAM INTERFACE PLANE WHICH ARE
C     TRANSMITTED THROUGH THE ELEMENT TO THE DOWNSTREAM
C     INTERFACE PLANE
C
      DO 90 IW = NUPSTRE+1,NWAVETYPE
       DO 100 IWP = NUPSTRE+1,NWAVETYPE
         WRITE(IUNIT,999)((((( S^DU_W'W(N, K, KP, MU, MUP),
   &       MU  = 0,            MUMAX(N,K,IW)),                    <- μ
   &       MUP = 0,            MUMAX(N,KP,IWP)),                  <- μ'
   &       K   = KMIN(N,IW), KMAX(N,IW)),                         <- k
   &       KP  = KMIN(N,IWP),KMAX(N,IWP)),                        <- k'
   &       N   = 1,            NHT)                               <- n
100     CONTINUE
 90   CONTINUE
C
C     WRITE SCATTERING MATRIX INFORMATION, S21, S31, S41, AT THE
C     INTERFACE PLANE DOWNSTREAM OF THIS REGION.  THESE SCATTERING
C     COEFFICIENTS CORRESPOND TO UPSTREAM PROPAGATING WAVES
C     REFLECTED BY THE ACOUSTIC ELEMENT AS DOWNSTREAM
C     PROPAGATING WAVES.
C
      DO 110 IW = 1,NUPSTRE
       DO 120 IWP = NUPSTRE+1,NWAVETYPE
         WRITE(IUNIT,999)((((( S^DD_W'W(N, K, KP, MU, MUP),
   &       MU  = 0,            MUMAX(N,K,IW)),                    <- μ
   &       MUP = 0,            MUMAX(N,KP,IWP)),                  <- μ'
   &       K   = KMIN(N,IW), KMAX(N,IW)),                         <- k
   &       KP  = KMIN(N,IWP),KMAX(N,IWP)),                        <- k'
   &       N   = 1,            NHT)                               <- n
120     CONTINUE
110   CONTINUE
```

I.3.4.3 Source Vector (see Section I.3.6)

Card 20: IFORCE = 0 No Source Vector is Input
= 1 Source Vector is input

IF IFORCE = 1:

Card 21: Write: FORMAT(1X,'Source Vector')

Card 22: Upstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = 1,NUPSTRE
    READ(IUNIT,999) ((( B_W^U(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),              ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                    ←k
  &    N   = 1,            NHT)                         ←n
10  CONTINUE
```

Card 23: Downstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = NUPSTRE + 1,NPRESDIR
    READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),              ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                    ←k
  &    N   = 1,            NHT)                         ←n
10  CONTINUE
```

Card 24: Downstream propagating vorticity waves normalized by c_∞.

```
   DO 10 IW = NPRESDIR + 1, NWAVETYPE
    READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),              ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                    ←k
  &    N   = 1,            NHT)                         ←n
10  CONTINUE
```

ENDIF

I.3.5 Aft Radiation Code: 20 ≤ ICODE < 30

I.3.5.1 Header Cards

Information in Cards 11 through 16 are only used to confirm for the user which Acoustic Properties File the user has chosen. These parameters are not used in code calculations. While parameters in Card 10 are not presently required for code calculations, this information expected to be required for code calculations in the future.

Card 10:

- SIGMAR: Hub to tip ratio of the duct at XINTER. This number should, strictly speaking, be the same for two codes meeting at a given interface plane.
- XINTER: Axial location of the interface plane relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- MINF: Far−field axial Mach number (far from the nacelle, positive downstream)
- MDUCT: Mass averaged axial Mach number at the interface plane (positive downstream)

Card 11: NLINO: Number of acoustically Lined Elements (outer wall). Set this value equal to zero if there is no outer wall liner.

If (NLINO .GT. 0) THEN

Card 12:

- MBEGO: Outer wall element number for start of liner
- XBEGO: Axial location where outer liner begins relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- XENDO: Axial Location where outer liner ends relative to the origin (non−dimensionalized by DFAN) as specified by the user.

Card 13: WRITE(IUNIT,999) ZIMPO(I),I=MBEGO,NLINO+MBEGO: Liner impedance (Resistance, Reactance) for each element on outer wall non−dimensionalized by local ρc (i.e. density x acoustic speed).

ENDIF

Card 14: NLINI: Number of acoustically lined elements (inner wall). Set this value equal to zero if there is no inner wall liner.

If (NLINI .GT. 0) THEN

Card 15:

- MBEGI: Inner wall element number for start of liner
- XBEGI: Axial location where inner liner begins relative to the origin (non−dimensionalized by DFAN) as specified by the user.
- XENDI: Axial Location where inner liner ends relative to the origin (non−dimensionalized by DFAN) as specified by the user.

Card 16: WRITE(IUNIT,999) ZIMPI(I),I=MBEGI,NLINI+MBEGI: Liner impedance (Resistance, Reactance) for each element on the inner wall non−dimensionalized by local ϱc (i.e. density x acoustic speed).

ENDIF

I.3.5.2 Scattering Coefficients (see Section I.3.6)

Card 17: Write: FORMAT(1X,'Scattering Matrix and Source Vector')

Card 18: Write: FORMAT(1X,'Scattering Matrix')

Card 19: Scattering coefficients as follows

```
C
C   WRITE SCATTERING MATRIX INFORMATION, S12, AT THE INTERFACE
C   PLANE UPSTREAM OF THIS REGION. THESE SCATTERING
C   COEFFICIENTS CORRESPOND TO DOWNSTREAM PROPAGATING WAVES
C   SCATTERED INTO UPSTREAM PROPAGATING WAVES. NOTE THAT
C   S13 & S14 ARE NOT WRITTEN SINCE THEY ARE VORTICITY WAVE BASED.
C   VORTICITY WAVES ARE NOT MODELED AS INPUT TO THE AFT
C   RADIATION CODES. THUS S13 AND S14 ARE ZERO.
C
      DO 10 IW = NUPSTRE+1,NPRESDIR
        DO 20 IWP = 1, NUPSTRE
C
          WRITE(IUNIT,999)(((( S^UU_W'W(N,K,MU,MUP),
     &       MUP  = 0,         MUMAX(N,K,IWP)),              <-μ'
     &       MU = 0,           MUMAX(N,K,IW)),               <-μ
     &       K  = KMIN(N,IW),KMAX(N,IW)),                    <-k
     &       N  = 1,           NHT)                          <-n
C
20      CONTINUE
10    CONTINUE
```

I.3.5.3 Source Vector (see Section I.3.6)

Card 20: IFORCE = 0 No Source Vector is Input
= 1 Source Vector is input

IF IFORCE = 1:

Card 21: Write: FORMAT(1X,'Source Vector')

Card 22: Upstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = 1,NUPSTRE
     READ(IUNIT,999) ((( B^U_W(MU,K,N),
  &  MU  = 0,          MUMAX(N,K,IW)),          <-μ
  &  K   = KMIN(N,IW),KMAX(N,IW)),              <-k
  &  N   = 1,          NHT)                     <-n
10 CONTINUE
```

Card 23: Downstream propagating pressure mode amplitudes normalized by PINF

```
   DO 10 IW = NUPSTRE + 1,NPRESDIR
     READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),            ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                  ←k
  &    N   = 1,            NHT)                       ←n
10   CONTINUE
```

Card 24: Downstream propagating vorticity waves normalized by c_∞.

```
   DO 10 IW = NPRESDIR + 1, NWAVETYPE
     READ(IUNIT,999) ((( B_W^D(MU,K,N),
  &    MU  = 0,            MUMAX(N,K,IW)),            ←μ
  &    K   = KMIN(N,IW),KMAX(N,IW)),                  ←k
  &    N   = 1,            NHT)                       ←n
10   CONTINUE
```

ENDIF

I.3.5.4 Far–field Directivities

Card 25: Write: FORMAT(1X,'Farfield Directivities')

- NANGLE: Number of polar angles (θ in Figure 5) in the directivity pattern. The code will interpolate these angles to form an array every 1^0 from $0^0 \leq$ ANGLE $\leq 180^0$. In the present code, any angle which is within 25^0 of the forward most computational boundary will be discarded. This is consistent with the baffle boundary condition which presently is found in the Eversman Aft Radiation code.
- DARRAY: Radial distance (R in Figure 5) from the origin non–dimensionalized by DFAN (origin specified by the user) of the polar directivity array.

Card 26: Far–field angle in degrees (0^0 is in front of the inlet)

- WRITE(IUNIT,*)(ANGLE(IANGLE), IANGLE = 1, NANGLE)

Card 27: Write: FORMAT(1X,'COMPLEX PRESSURE DIRECTIVITIES')

Card 28: Complex pressure directivities (real and imaginary parts of the directivities for each mode):

```
      DO 10 IW = NUPSTRE + 1,NPRESDIR
   C
        WRITE(IUNIT,999) (((( DIRECTIN(N,K,MU,IANGLE),
          IANGLE = 1,           NANGLE),
          MU     = 0,           MUMAX(N,K,IW)),
          K      = KMIN(N,IW), KMAX(N,IW))
          N      = 1, NHT)
   10  CONTINUE
```

I.3.6 Format for Scattering Coefficient and Source Vector Output

Section 3.2 discusses the conventions for the scattering coefficients and source vector information in CUP3D. Further discussion of the "Kinematics of Mode Scattering" may also be found in Reference 1. The discussion in this section will center on how scattering coefficients and source vectors are non–dimensionalized and represented in CUP3D.

A two sided Fourier transform is utilized where the complex scattering coefficient and source vector amplitudes represent values of the coefficients in this series. Equations (20) through (25) in Section 3.2 give further details. Only the positive harmonics of the source vectors and scattering coefficients are input. Because the code works with tone noise, the negative harmonics are assumed by the code to be the complex conjugates of the positive harmonics with the exception of rotor scattering. In the rotor case, positive harmonics scatter into negative harmonics thus requiring scattering coefficients to be input.

All scattering coefficients, source vectors and far–field directivities are read in with format given by:

999 FORMAT(10E13.6)

where this format will include five coefficients (i.e. real and imaginary parts) per line.

The scattering coefficients, S, are effectively the reflection and transmission coefficients for each acoustic element. The scattering coefficients take on the form $S^{P'P}_{W'W}$ where the scattering coefficient subscripts denote the wave type, W, being scattered into the wave type, W'. The superscripts denote interface plane, P, scattering into the interface plane, P'.

All source vector information should be non–dimensionalized by:

- The ambient pressure, PINF for pressures waves,
- The ambient acoustic speed, $c_\infty = \sqrt{\gamma_a R_a T_\infty}$ where T_∞ = TINF, for vorticity waves.

TINF and PINF are defined as the static temperature and pressure far from the nacelle.

In the present code, the following wave types are in use:

$W = 1$ Upstream going pressure waves
$W = 2$ Downstream going pressure waves
$W = 3$ "no–w" vorticity wave (downstream only)
$W = 4$ "no–u" vorticity wave (downstream only)

In Sections I.3.2 to I.3.5, FORTRAN comment statements are shown which refer only to these four wave types. However, the number of wave types can be changed by changing NUPSTRE, NPRESDIR, and/or NWAVETYPE in the parameter statements in the CUP3D code.

For scattering coefficient and source vector input formats discussed in Sections I.3.2 to I.3.5, the superscripts (P and P') are given by:

U = interface plane upstream of acoustic element = IUPSTR(IREGION) in CUP3D System File

D = interface plane downstream of acoustic element = IDWNST(IREGION) in CUP3D System File

For scattering coefficient and source vector input formats discussed in Sections I.3.2 to I.3.5 the subscripts (W and W') are given by:

W = IW = Wave type of the wave travelling into the acoustic element i.e. right hand side of Equation (15)

W' = IWP = Wave type for the wave leaving the acoustic element (i.e. the scattered wave) i.e. left hand side of Equation (15)

NUPSTRE = Number of upstream travelling pressure wave types (= 1 in the present code)

NPRESDIR = Total number of pressure wave types (= 2 in the present code)

NWAVETYPE = Total number of wave types: pressure + vorticity (= 4 in the present code)

I.3.7 Examples of Acoustic Properties Files

Example 1: Inlet Radiation Code (same style as aft radiation code)

```
    11  1
  Case Number  1
 Test run for inlet radiation code
    2    723.02709736824
    16  22
    0  1  0  2  0  1  0  2  0  1  0  2  0  1  0  2
    16  -6  32  10  -12  16  -6  32  10  -12  16  -6  32  10  -12  16  -6  32  10
    -12
    0  2  0  4  3  0  2  0  4  3  0  2  0  4  3  0  2  0  4  3
  0.475223E+01-0.151888E+02 0.140535E+02 0.000000E+00 0.475223E+01-0.792470E+00 0.475223E+01-0.103175E+02 0.950446E+01-0.281977E+02
  0.306418E+02 0.000000E+00 0.266233E+02 0.000000E+00 0.215012E+02 0.000000E+00 0.950446E+01-0.499092E+01 0.950446E+01-0.164505E+02
  0.290826E+02 0.000000E+00 0.235486E+02 0.000000E+00 0.133733E+02 0.000000E+00 0.950446E+01-0.130216E+02 0.475223E+01 0.151888E+02
 -0.454904E+01 0.000000E+00 0.475223E+01 0.792470E+00 0.475223E+01 0.103175E+02 0.950446E+01 0.281977E+02-0.116329E+02 0.000000E+00
 -0.761440E+01 0.000000E+00-0.249231E+01 0.000000E+00 0.950446E+01 0.499092E+01 0.950446E+01 0.164505E+02-0.100737E+02 0.000000E+00
 -0.453972E+01 0.000000E+00 0.563562E+01 0.000000E+00 0.950446E+01 0.130216E+02
     0.44327000000000    -6.6046173454609D-02    0.20000000000000    0.38500000000000
    0
    0
   Scattering Matrices and Source Vector
   Scattering Matrix
 -0.365344E-02-0.446337E-02-0.102875E-02 0.522877E-01-0.160511E+00-0.304812E-02 0.146651E-01-0.141137E-01 0.470927E-02-0.427195E-02
 -0.513261E+00-0.230566E-01 0.271421E-02 0.760939E-02 0.142889E-02-0.199256E-03 0.490851E-01-0.290320E-01-0.128391E-01-0.895564E-02
 -0.555029E-02-0.605181E-02-0.116662E-01-0.797645E-03-0.797289E-02-0.528578E-02 0.227633E-01 0.228062E-01-0.372404E-01 0.909213E-02
  0.108616E-01-0.628020E-02-0.910079E-02 0.182574E-02-0.969254E-02 0.811641E-02 0.357407E-01-0.379178E-01 0.580331E-01-0.172466E-01
 -0.104490E-01 0.102368E-01 0.123704E-01 0.286756E-03 0.163801E-01-0.106996E-01 0.390604E-01 0.219257E-01-0.105778E+00 0.734644E-01
  0.726195E-02-0.207893E-01-0.112431E-02-0.180647E-02 0.233724E-03-0.306362E-02-0.153825E-02 0.131896E-02-0.150012E+00-0.195508E-01
  0.119859E-03 0.137132E-01 0.683015E-03 0.168428E-02-0.212172E-02-0.194118E-03-0.505330E-03-0.699074E-02 0.298327E-01-0.352612E-01
 -0.217939E-01-0.114226E-01 0.431427E-02 0.102868E-01 0.600097E-02-0.335784E-01 0.559724E-01 0.479707E-01-0.136671E-01 0.825091E-02
 -0.542824E-02-0.171375E-01-0.978956E-02 0.286509E-01-0.116975E+00-0.189022E+00 0.191952E-01-0.257958E-01 0.371782E-02-0.238809E-02
  0.165858E-01-0.280431E-02 0.358698E+00-0.797814E+00 0.372118E-01 0.235190E-01-0.497529E-03-0.146018E-02 0.300783E-02-0.354689E-02
  0.109870E+00 0.328976E-01-0.290999E-01-0.867662E-02
    0
   Farfield Directivities
   488    5.0002103006778
    0.    0.32796468222335    0.65592936444669    0.98389404667004    1.3118587288934
      1.6398234111167    1.9677880933401    2.2957527755634    2.6237174577868
      2.9516821400101    3.2796468222335    3.6075673655363    3.9354879088392
      4.2634084521421    4.5913289954450    4.9192495387478    5.2471700820507
      5.5750906253536    5.9030111686564    6.2309317119593    6.5588522552622
      6.8398585500904    7.1208648449186    7.4018711397468    7.6828774345750
      7.9638837294032    8.2448900242314    8.5258963190596    8.8069026138878
      9.0879089087160    9.3689152035442    9.6498293714417    9.9307435393391
     10.2116577072366   10.4925718751340   10.773486043031    11.054400210929
      11.335314378826    11.616228546724    11.897142714621    12.178056882519
      12.458851805329    12.739646728138    13.020441650948    13.301236573758
      13.582031496568    13.862826419378    14.143621342187    14.424416264997
```

14.705211187807	14.986006110617	15.219891400852	15.453776691088
15.687661981323	15.921547271558	16.155432561794	16.389317852029
16.623203142265	16.857088432500	17.090973722736	17.324859012971
17.558627758184	17.792396503397	18.026165248610	18.259933993824
18.493702739037	18.727471484250	18.961240229463	19.195008974676
19.428777719889	19.662546465103	19.896183572454	20.129820679806
20.363457787158	20.597094894510	20.830732001862	21.064369109214
21.298006216565	21.531643323917	21.765280431269	21.998917538621
22.232408420080	22.465899301540	22.699390183000	22.932881064459
23.166371945919	23.399862827378	23.633353708838	23.866844590297
24.100335471757	24.333826353216	24.520504143593	24.707181933969
24.893859724346	25.080537514722	25.267215305099	25.453893095475
25.640570885851	25.827248676228	26.013926466604	26.200604256981
26.387172162221	26.573740067460	26.760307972700	26.946875877940
27.133443783179	27.320011688419	27.506579593659	27.693147498898
27.879715404138	28.066283309378	28.252734267357	28.439185225336
28.625636183315	28.812087141294	28.998538099273	29.184989057252
29.371440015232	29.557890973211	29.744341931190	29.930792889169
30.117120066628	30.303447244087	30.489774421545	30.676101599004
30.862428776463	31.048755953922	31.235083131380	31.421410308839
31.607737486298	31.794064663757	31.980261383318	32.166458102879
32.352654822440	32.538851542001	32.725048261562	32.911244981123
33.097441700684	33.283638420245	33.469835139806	33.656031859367
33.842091689286	34.028151519206	34.214211349125	34.400271179044
34.586331008963	34.772390838883	34.958450668802	35.144510498721
35.330570328641	35.516630158560	35.702546770670	35.888463382780
36.074379994891	36.260296607001	36.446213219111	36.632129831221
36.818046443331	37.003963055442	37.189879667552	37.375796279662
37.561563612614	37.747330945566	37.933098278518	38.118865611470
38.304632944422	38.490400277374	38.676167610326	38.861934943278
39.047702276230	39.233469609181	39.419081732327	39.604693855472
39.790305978618	39.975918101763	40.161530224909	40.347142348054
40.532754471199	40.718366594345	40.903978717490	41.089590840636
41.275042070362	41.460493300088	41.645944529814	41.831395759540
42.016846989266	42.202298218992	42.387749448718	42.573200678444
42.758651908170	42.944103137896	43.129388088380	43.314673038865
43.499957989349	43.685242939833	43.870527890317	44.055812840802
44.241097791286	44.426382741770	44.611667692255	44.796952642739
44.982065956870	45.167179271001	45.352292585131	45.537405899262
45.722519213393	45.907632527524	46.092745841655	46.277859155786
46.462972469917	46.648085784047	46.819189716165	46.990293648283
47.161397580401	47.332501512519	47.503605444637	47.674709376755
47.845813308873	48.016917240991	48.188021173109	48.359125105227
48.524076004468	48.689026903710	48.853977802952	49.018928702194
49.183879601436	49.348830500678	49.513781399920	49.678732299162
49.843683198404	50.008634097646	50.170894382505	50.333154667365
50.495414952225	50.657675237084	50.819935521944	50.982195806804
51.144456091664	51.306716376523	51.468976661383	51.631236946243
51.793983352164	51.956729758086	52.119476164007	52.282222569928
52.444968975849	52.607715381771	52.770461787692	52.933208193613
53.095954599535	53.258701005456	53.506753892494	53.754806779533

54.002859666571	54.250912553609	54.498965440648	54.747018327686
54.995071214725	55.243124101763	55.491176988801	55.739229875840
55.993880782079	56.248531688319	56.503182594558	56.757833500798
57.012484407037	57.267135313276	57.521786219516	57.776437125755
58.031088031995	58.285738938234	58.548143552963	58.810548167692
59.072952782421	59.335357397149	59.597762011878	59.860166626607
60.122571241336	60.384975856064	60.647380470793	60.909785085522
61.180493495290	61.451201905058	61.721910314826	61.992618724594
62.263327134363	62.534035544131	62.804743953899	63.075452363667
63.346160773435	63.616869183203	63.990967990549	64.365066797894
64.739165605239	65.113264412584	65.487363219929	65.861462027274
66.235560834619	66.609659641964	66.983758449309	67.357857256654
67.746670345441	68.135483434228	68.524296523014	68.913109611801
69.301922700588	69.690735789375	70.079548878162	70.468361966949
70.857175055736	71.245988144523	71.648412707496	72.050837270470
72.453261833444	72.855686396417	73.258110959391	73.660535522365
74.062960085339	74.465384648312	74.867809211286	75.270233774260
75.684612698861	76.098991623462	76.513370548063	76.927749472664
77.342128397266	77.756507321867	78.170886246468	78.585265171069
78.999644095671	79.414023020272	79.838764314120	80.263505607968
80.688246901816	81.112988195664	81.537729489512	81.962470783360
82.387212077209	82.811953371057	83.236694664905	83.661435958753
84.093266794148	84.525097629543	84.956928464938	85.388759300333
85.820590135728	86.252420971123	86.684251806519	87.116082641914
87.547913477309	87.979744312704	88.414087236386	88.848430160069
89.282773083751	89.717116007434	90.151458931116	90.585801854799
91.020144778481	91.454487702164	91.888830625846	92.323173549528
92.755538370142	93.187903190755	93.620268011369	94.052632831982
94.484997652595	94.917362473209	95.349727293822	95.782092114435
96.214456935049	96.646821755662	97.073046667534	97.499271579406
97.925496491278	98.351721403150	98.777946315022	99.204171226894
99.630396138766	100.056621050638	100.482845962510	100.909070874382
101.325541654644	101.742012434906	102.158483215169	102.574953995431
102.991424775693	103.407895555955	103.824366336217	104.240837116479
104.657307896742	105.073778677004	105.47751993470	105.88126119240
106.28500245010	106.68874370780	107.09248496550	107.49622622320
107.89996748090	108.30370873860	108.70744999630	109.11119125400
109.49998824297	109.88878523194	110.27758222091	110.66637920988
111.05517619885	111.44397318782	111.83277017679	112.22156716576
112.61036415474	112.99916114371	113.37145990016	113.74375865662
114.11605741308	114.48835616953	114.86065492599	115.23295368244
115.60525243890	115.97755119536	116.34984995181	116.72214870827
117.07704767361	117.43194663894	117.78684560428	118.14174456962
118.49664353496	118.85154250030	119.20644146563	119.56134043097
119.91623939631	120.27113836165	120.60824814643	120.94535793122
121.28246771600	121.61957750079	121.95668728557	122.29379707035
122.63090685514	122.96801663992	123.30512642471	123.64223620949
123.96162445458	124.28101269967	124.60040094476	124.91978918985
125.23917743494	125.55856568003	125.87795392512	126.19734217021
126.51673041531	126.83611866040	127.13812357497	127.44012848954
127.74213340411	128.04413831869	128.34614323326	128.64814814783

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      128.95015306240     129.25215797697     129.55416289155     129.85616780612
      130.14136310295     130.42655839977     130.71175369660     130.99694899343
      131.28214429025     131.56733958708     131.85253488391     132.13773018074
      132.42292547756     132.70812077439     132.97720379680     133.24628681921
      133.51536984163     133.78445286404     134.05353588645     134.32261890886
      134.59170193127     134.86078495369     135.12986797610     135.39895099851
      135.65271199100     135.90647298349     136.16023397598     136.41399496846
      136.66775596095     136.92151695344     137.17527794593
   COMPLEX PRESSURE DIRECTIVITIES
 -0.189120E-20 0.512816E-21-0.590248E-21-0.278571E-21 0.632164E-21-0.669726E-21 0.177492E-20-0.658018E-21 0.283492E-20-0.241415E-21
  0.380703E-20 0.581500E-21 0.468416E-20 0.181149E-20 0.545718E-20 0.344859E-20 0.611502E-20 0.549210E-20 0.664463E-20 0.794045E-20
  0.703105E-20 0.107911E-19 0.253732E-20-0.293418E-20-0.232209E-20-0.117748E-19-0.795872E-20-0.157035E-19-0.145134E-19-0.147745E-19
 -0.221497E-19-0.905171E-20-0.310536E-19 0.138916E-20-0.414322E-19 0.164597E-19-0.535132E-19 0.360572E-19-0.675438E-19 0.600628E-19
 -0.837890E-19 0.883400E-19-0.150492E-20-0.709252E-19 0.779305E-19-0.142686E-18 0.142775E-18-0.131478E-18 0.189173E-18-0.383009E-19
  0.213026E-18 0.135632E-18 0.210004E-18 0.388868E-18 0.175563E-18 0.719693E-18 0.104959E-18 0.112611E-17-0.672785E-20 0.160581E-17
 -0.164576E-18 0.215616E-17 0.427326E-18 0.682139E-18 0.605464E-18 0.470649E-18 0.162098E-18 0.142487E-17-0.976592E-18 0.349557E-17
 -0.288574E-17 0.662763E-17-0.564130E-17 0.107597E-16-0.931955E-17 0.158240E-16-0.139965E-16 0.217461E-16-0.197473E-16 0.284442E-16
 -0.266457E-16 0.358297E-16-0.351384E-16 0.268506E-16-0.470227E-16 0.206538E-16-0.637130E-16 0.167073E-16-0.853770E-16 0.145445E-16
 -0.112161E-15 0.136720E-16-0.144187E-15 0.135704E-16-0.181549E-15 0.136956E-16-0.224308E-15 0.134798E-16-0.272492E-15 0.123334E-16
 -0.326091E-15 0.964652E-17-0.370245E-15-0.222672E-16-0.414167E-15-0.660262E-16-0.459141E-15-0.122477E-15-0.504130E-15-0.191814E-15
 -0.548054E-15-0.274177E-15-0.589800E-15-0.369646E-15-0.628220E-15-0.478236E-15-0.662135E-15-0.599895E-15-0.690341E-15-0.734497E-15
 -0.711608E-15-0.881839E-15-0.720406E-15-0.102522E-14-0.682516E-15-0.120971E-14-0.590529E-15-0.142919E-14-0.441090E-15-0.167888E-14
etc.
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Example 2: Stator (same style as the rotor) with 2 cases

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   41   2
 Case Number  1
Test Run for Stator Output
   3    406.67916900720
   16   22
   1  0  1  1  2  2  1  0  1  1  2  2  1  0  1  1  2  2  1  0  1  1  2  2
   10  4  10  4  10  4  10  4
   0  3  0  3  0  3  0  3
-0.652887E+00 0.000000E+00-0.259324E+01 0.000000E+00-0.241422E+01 0.000000E+00-0.203149E+01 0.000000E+00-0.638075E+00 0.000000E+00
-0.519879E-01 0.000000E+00 0.153593E+01 0.000000E+00 0.135691E+01 0.000000E+00 0.974182E+00 0.000000E+00-0.419237E+00 0.000000E+00
    0.44264194669757    0.22821552723059    1.0000000000000    0.24500000000000
    0.36466402395932    1.0084604712063    0.96553814105035
    0.44264194669757    0.53597914252607    1.0000000000000    0.24500000000000
    0.36466402395932    1.0084604712063    0.96553814105035
   15
    0.44264194669757    0.48860254924681    0.52944727694090    0.56670104287370
    0.60122132097335    0.66422132097335    0.72121784472769    0.77382618771727
    0.82315295480881    0.86995365005794    0.91479721900348    0.93662456546929
    0.95810196987254    0.97922943221321    1.0000000000000
     2.7068731079569     2.3497479928425     2.1442204073127     1.9817425245212
     1.8539069010869     1.6607140833557     1.5183491678466     1.4076374971045
     1.3180761072857     1.2435907703557     1.1804608836082     1.1521324045552
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 1.1258269758192     1.1013128447391     1.0777969057772
 53.364200000000     59.272500000000     61.637900000000     62.768100000000
 63.423200000000     64.431800000000     65.095800000000     65.510900000000
 65.791500000000     65.655700000000     65.322400000000     65.141500000000
 64.858200000000     64.405400000000     64.059000000000
 2.3176894647057     2.3087470864849     2.3000130767286     2.2887374947598
 2.2803400737985     2.2486899574842     2.2379228726165     2.2554382251117
 2.2962475452649     2.3670808302993     2.4815041621135     2.5581373929486
 2.6536931930396     2.7805093615952     2.9875860709216
Scattering Matrix and Source Vector
Scattering Matrix
 0.123900E+01-0.476344E+00 0.384587E-01 0.165659E-02-0.117480E-01 0.650623E-02-0.157748E-01 0.781025E-02-0.310022E-01 0.154245E+00
-0.146535E-01 0.862793E-02-0.767622E-02 0.140633E-01 0.406166E-01 0.259637E-01 0.845616E-01 0.954972E-01-0.336928E-02-0.584703E-02
 0.172368E-01 0.658938E-02 0.103217E-01-0.926706E-02-0.877353E-01 0.216844E+00-0.178349E+00 0.222776E+00 0.663221E-01 0.713950E-01
 0.371811E+00 0.860204E+00-0.638068E+01-0.114066E+00
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-0.300496E+00 0.786848E+00 0.192457E+00 0.268622E+00-0.170017E+00-0.439880E+00-0.572146E-01 0.154924E+00-0.694474E-01-0.157301E+00
-0.550598E+00-0.518508E+00-0.622824E+00-0.293669E+00-0.333930E+00 0.781194E-01-0.134786E+00-0.334855E-01 0.372559E+00 0.215014E+00
-0.218099E+00 0.516284E+00-0.361391E+00 0.274112E+00-0.975123E-02-0.266787E+00 0.443352E+01-0.285035E+01 0.137435E+01-0.232895E+01
 0.110585E+01 0.288230E+01 0.464135E+01-0.372447E+01
-0.561757E+00 0.158156E+00-0.305239E+00 0.943657E+00 0.269188E-02 0.157519E-02-0.314103E-03 0.243691E-02 0.526639E-02 0.274210E-02
 0.296098E-02 0.430281E-02 0.597306E+00 0.800000E+00 0.234221E-02 0.448968E-02 0.251737E-02-0.121277E-02-0.736891E-02 0.294452E-03
-0.412893E-02 0.810580E-02 0.477213E+00-0.869977E+00 0.328254E-01 0.451358E-01 0.175830E+00-0.215854E+00-0.607237E-01-0.682444E-01
-0.281238E+00-0.840574E+00 0.547620E+01-0.460712E+00
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-0.354925E+00-0.114814E+01-0.178581E-01 0.159707E-01 0.460757E-02-0.511673E-02-0.308382E-03 0.193578E-02-0.594565E-02 0.467806E-03
 0.944709E-02-0.106446E-01 0.490429E-02-0.924726E-02-0.129416E-02-0.743528E-02-0.152640E-02 0.241508E-02 0.335768E-01 0.255792E-02
 0.174965E-02-0.569937E-01-0.149489E-02-0.102350E-01-0.547825E-01-0.178361E-01-0.412044E+01 0.224722E+01-0.136451E+01 0.197439E+01
-0.125040E+01-0.234646E+01-0.538493E+01 0.312505E+01
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  Source Vector
 0.927498E-04-0.164002E-03 0.300982E-05 0.455360E-05 0.144927E-04 0.545106E-05-0.152271E-04-0.128581E-04-0.187828E-03-0.562735E-03
-0.218285E-03 0.132318E-03 0.337759E-06 0.371304E-06-0.103058E-06 0.100681E-05 0.121366E-05-0.445164E-05 0.916513E-04 0.536424E-03
 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
 Case Number  2
Test run for Stator Output
   3    451.86574334133
   16  22
   1  0  1  1  2  3  1  0  1  1  2  3  1  0  1  1  2  3  1  0  1  1  2  3
   10  4  -18  10  4  -18  10  4  -18  10  4  -18
   0  3  0  0  3  0  0  3  0  0  3  0
-0.120377E+01 0.000000E+00-0.297628E+01 0.000000E+00-0.281807E+01 0.000000E+00-0.249318E+01 0.000000E+00-0.171972E+01 0.000000E+00
-0.697885E+00 0.000000E+00 0.351664E+00 0.000000E+00 0.169812E+01 0.000000E+00 0.153991E+01 0.000000E+00 0.121502E+01 0.000000E+00
 0.441557E+00 0.000000E+00-0.580278E+00 0.000000E+00
    0.44264194669757    0.22821552723059    1.0000000000000    0.26400000000000
    0.40489915062853     1.0121453591349    0.96688878691820
    0.44264194669757    0.53597914252607    1.0000000000000    0.26400000000000
    0.40489915062853     1.0121453591349    0.96688878691820
   15
    0.44264194669757    0.48860254924681    0.52944727694090    0.56670104287370
    0.60122132097335    0.66422132097335    0.72121784472769    0.77382618771727
    0.82315295480881    0.86995365005794    0.91479721900348    0.93662456546929
    0.95810196987254    0.97922943221321    1.0000000000000
     2.7068731079569     2.3497479928425     2.1442204073127     1.9817425245212
     1.8539069010869     1.6607140833557     1.5183491678466     1.4076374971045
```

```
     1.3180761072857     1.2435907703557     1.1804608836082     1.1521324045552
     1.1258269758192     1.1013128447391     1.0777969057772
     53.364200000000     59.272500000000     61.637900000000     62.768100000000
     63.423200000000     64.431800000000     65.095800000000     65.510900000000
     65.791500000000     65.655700000000     65.322400000000     65.141500000000
     64.858200000000     64.405400000000     64.059000000000
     2.3243420882584     2.3165056041266     2.3086280138673     2.2977062361747
     2.2895999221815     2.2579725073958     2.2478601133944     2.2656013677883
     2.3067404437395     2.3777407031063     2.4921244425289     2.5690370277864
     2.6643313194453     2.7918849882969     2.9994013627572
  Scattering Matrix and Forcing Function Arrays
  Scattering Matrix
-0.211221E+00-0.119433E+00-0.227465E-01 0.210265E-01-0.838865E-03-0.644849E-02 0.393785E-02 0.289299E-02-0.300379E-01 0.140781E-01
-0.509313E-02-0.625566E-02 0.649997E-02 0.630174E-02-0.165586E-01-0.860256E-03 0.288431E-01 0.354949E-01 0.939128E-02-0.377667E-02
etc.
.
.
.
```

APPENDIX II: POWER CALCULATION IN CUP3D COUPLED CASCADE CODE

The objective of this appendix is to develop the equations being used to calculate power in CUP3D. The sound power level is calculated at interface planes just upstream and just downstream of the noise source. The following assumptions are made at those interface planes:

- Locally constant area annular duct,
- Locally uniform axial flow

As a result, for the sound power calculation, the standard Tyler/Sofrin modes (Reference 13) are used. A more general formulation may be developed in the future.

From Equation (4), the perturbation pressure is assumed to be of the form:

$$p_W^P(X,r,\phi,t) = p_\infty \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} A_{Wnk\mu}^P \psi_{Wm\mu}^P(r) e^{i\left[m\phi + \gamma_{Wnk\mu}^P\left(X - X_W^P\right) - nB\Omega t\right]} \tag{II.1}$$

Perturbation velocity may be related to perturbation pressure by:

$$u_W^P(X,r,\phi,t) = c_\infty \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} R_{Wnk\mu}^P A_{Wnk\mu}^P \psi_{Wm\mu}^P(r) e^{i\left[m\phi + \gamma_{Wnk\mu}^P\left(X - X_W^P\right) - nB\Omega t\right]} \tag{II.2}$$

$R_{Wnk\mu}^P$ may then be found using the small perturbation, unsteady axial momentum equation for a uniform axial mean flow (consistent with Equation C.11 in Reference 11):

$$\frac{\partial u}{\partial t} + U_P \frac{\partial u}{\partial x} + \frac{1}{\rho_P}\frac{\partial p}{\partial x} = 0 \tag{II.3}$$

Substituting Equations (II.1) and (II.2) into Equation (II.3) we get:

$$ic_\infty\left(-nB\Omega + U_P\gamma_{Wnk\mu}^P\right)R_{Wnk\mu}^P = -i\frac{p_\infty}{\rho_P}\gamma_{Wnk\mu}^P \tag{II.4}$$

Multiply through by $-\frac{r_{d_P}}{c_P}$ and solve for $R_{Wnk\mu}^P$ to get:

$$R_{Wnk\mu}^P = \frac{p_\infty}{c_\infty \rho_P c_P}\hat{R}_{Wnk\mu}^P \tag{II.5}$$

Where:

$$\hat{R}_{Wnk\mu}^P = \frac{\hat{\gamma}_{Wnk\mu}^P}{\left(nBM_{t_P} - M_{x_P}\hat{\gamma}_{Wnk\mu}^P\right)} \tag{II.6}$$

Note that for propagating waves, $R_{Wnk\mu}^P$ is real. This expression is consistent with Equation C.14 in Reference 11 except that Reference 11 non-dimensionalizes differently.

The equation for time dependent intensity in a uniform axial flow is used i.e. Equation C.4 in Reference 11 or Equation 101 in Reference 1:

$$I_P(X,r,\phi,t) = \left(1 + M_{x_P}^2\right)p_W^P u_W^P + \frac{M_{x_P}}{\rho_P c_P}\left(p_W^P\right)^2 + \rho_P c_P M_{x_P}\left(u_W^P\right)^2 \tag{II.7}$$

Upstream acoustic power will be calculated at the interface plane, $P = 1$ using upstream propagating acoustic waves (for a uniform axial flow there is one upstream going acoustic pressure wave type: $W = 1$). Downstream acoustic power needs to be calculated at the last interface plane, $P = N_p$, using downstream acoustic pressure waves (for a uniform axial flow there is one downstream going acoustic pressure wave type: $W = 2$). Acoustic power is expressed by:

$$\Pi_P = \int_{r_h}^{r_d}\int_0^{2\pi} \bar{I}_P \bar{r} d\phi d\bar{r} = r_d^2 \int_{\sigma}^{1}\int_0^{2\pi} \bar{I}_P r d\phi dr \tag{II.8}$$

where $\bar{r}$ is the dimensional radius and $\bar{I}_P$ is the time average intensity of I_P. To get $\bar{I}_P$, realize that only cuton modes contribute to the sound power level. In Equations (II.1) and (II.2) the perturbation pressure, p_W^P, and perturbation velocity, u_W^P are real. This means that $p_W^P = p_W^{P^*}$ and $u_W^P = u_W^{P^*}$. If N_h is given as the number of BPF harmonics being calculated and only cuton modes are included in the summation, then substitution of Equations (II.1) and (II.2) into Equation (II.7) leads to:

$$I_P(X,r,\phi,t) = \sum_{n=-N_h}^{N_h}\sum_k\sum_\mu\sum_{n'=-N_h}^{N_h}\sum_{k'}\sum_{\mu'} \psi_{m\mu}\psi_{m'\mu'}e^{i(\gamma-\gamma')(X-X_P)}e^{-i(n-n')B\Omega t}e^{i(m-m')\phi}A_{Wnk\mu}^P A_{Wn'k'\mu'}^{P^*} I_{1nk\mu n'k'\mu'} \tag{II.9}$$

where:

$$I_{1nk\mu n'k'\mu'} = \left[\left(1 + M_{x_P}^2\right)p_\infty c_\infty R_{Wn'k'\mu'}^{P^*} + \frac{M_{x_P}}{\rho_P c_P}p_\infty^2 + \rho_P c_P M_{x_P} c_\infty^2 R_{Wnk\mu}^P R_{Wn'k'\mu'}^{P^*}\right] \tag{II.10}$$

To get the time average intensity at interface plane, P, note that:

$$\overline{e^{-i(n-n')B\Omega t}} = \delta_{nn'} \tag{II.11}$$

so that Equation (II.9) becomes:

$$\bar{I}_P\left(X_P,r,\phi\right) = \sum_{n=-N_h}^{N_h}\sum_k\sum_\mu\sum_{k'}\sum_{\mu'} A_{Wnk\mu}^P A_{Wnk'\mu'}^{P^*} I_{1nk\mu k'\mu'}\psi_{m\mu}\psi_{m'\mu'}e^{i(\gamma-\gamma')(X-X_P)}e^{i(k-k')V\phi} \tag{II.12}$$

where the subscripts on I_1 in Equation (II.10) has been contracted in the obvious way. Substitute this equation into Equation (II.8):

$$\Pi_P = r_d^2 \sum_{n=-N_h}^{N_h}\sum_k\sum_\mu\sum_{k'}\sum_{\mu'} A_{Wnk\mu}^P A_{Wnk'\mu'}^{P^*} I_{1nk\mu k'\mu'} e^{i(\gamma-\gamma')(X-X_P)} \int_{\sigma}^{1}\int_0^{2\pi} \psi_{m\mu}\psi_{m'\mu'}e^{i(k-k')V\phi} r d\phi dr \tag{II.13}$$

The integral over ϕ yields a Kronecker delta:

$$\int_0^{2\pi} e^{-i(k-k')V\phi} d\phi = 2\pi\delta_{kk'} \tag{II.14}$$

so that:

$$\Pi_P = 2\pi r_d^2 \sum_{n=-N_h}^{N_h} \sum_k \sum_\mu \sum_{\mu'} A_{Wnk\mu}^P A_{Wnk\mu'}^{P^*} I_{1nk\mu\mu'} e^{i(\gamma-\gamma')(X-X_P)} \int_\sigma^1 \psi_{m\mu}\psi_{m\mu'} r dr \tag{II.15}$$

The orthogonality condition for the radial eigenmodes is:

$$\int_\sigma^1 \psi_{m\mu}\psi_{m\mu'} r dr = \delta_{\mu\mu'} C_{m\mu} \tag{II.16}$$

Thus when $\mu' = \mu$:

$$\int_\sigma^1 \psi_{m\mu}^2 r dr = C_{m\mu} \tag{II.17}$$

This integral is evaluated in Appendix III to determine the value of $C_{m\mu}$. In addition, since negative harmonics are equal to positive harmonics, use positive harmonics only and multiply Equation (II.15) by 2. The Kronecker delta in Equation (II.16) enables the μ' summation and eliminates the final exponential in Equation (II.15) since all of the implied subscripts on γ and γ' are now equal. Therefore Equation (II.15) becomes:

$$\Pi_P = 4\pi r_d^2 \sum_{n=1}^{N_h} \sum_k \sum_\mu A_{Wnk\mu}^P A_{Wnk\mu}^{P^*} I_{1nk\mu} C_{m\mu} \tag{II.18}$$

Equation (II.10) can now be substituted back into Equation (II.18):

$$\Pi_P = 4\pi r_d^2 \sum_{n=1}^{N_h} \sum_k \sum_\mu A_{Wnk\mu}^P A_{Wnk\mu}^{P^*} \left[\left(1 + M_{x_P}^2\right) p_\infty c_\infty R_{Wnk\mu}^{P^*} + \frac{M_{x_P}}{\rho_P c_P} p_\infty^2 + \rho_P c_P M_{x_P} c_\infty^2 R_{Wnk\mu}^P R_{Wnk\mu}^{P^*} \right] C_{m\mu} \tag{II.19}$$

Non-dimensionalize power by $p_\infty c_\infty \left(2r_d\right)^2$ so that:

$$\hat{\Pi}_P = \pi \sum_{n=1}^{N_h} \sum_k \sum_\mu A_{Wnk\mu}^P A_{Wnk\mu}^{P^*} \left[\left(1 + M_{x_P}^2\right) R_{Wnk\mu}^{P^*} + \frac{M_{x_P} p_\infty}{\rho_P c_P c_\infty} + \rho_P c_P M_{x_P} \frac{c_\infty}{p_\infty} R_{Wnk\mu}^{P^*} R_{Wnk\mu}^{P^*} \right] C_{m\mu} \tag{II.20}$$

Substitute Equation (II.5) into Equation (II.20) to get:

$$\hat{\Pi}_P = \pi \frac{p_\infty}{\rho_p c_p c_\infty} \sum_{n=1}^{N_h} \sum_k \sum_\mu A^P_{Wnk\mu} A^{P^*}_{Wnk\mu} \left[(1 + M^2_{x_P}) \hat{R}^P_{Wnk\mu} + M_{x_P} + M_{x_P} \hat{R}^P_{Wnk\mu} \hat{R}^{P^*}_{Wnk\mu} \right] C_{m\mu} \tag{II.21}$$

Since the γ's are real for propagating modes, Equation (II.6) shows that $\hat{R}^{P^*}_W = \hat{R}^P_W$ and Equation (II.21) becomes:

$$\hat{\Pi}_P = \pi \frac{p_\infty}{\rho_p c_p c_\infty} \sum_{n=1}^{N_h} \sum_k \sum_\mu \left| A^P_{Wnk\mu} \right|^2 \left[(1 + M^2_{x_P}) \hat{R}^P_{Wnk\mu} + M_{x_P} + M_{x_P} \left(\hat{R}^P_{Wnk\mu} \right)^2 \right] C_{m\mu} \tag{II.22}$$

Since $c^2 = \frac{\gamma_a P}{\rho} = \gamma_a R_a T$ where P = mean static pressure, Equation (II.22) can be written:

$$\hat{\Pi}_P = \pi \frac{p_\infty}{\gamma_a p_P} \sqrt{\frac{T_P}{T_\infty}} \sum_{n=1}^{N_h} \sum_k \sum_\mu \left| A^P_{Wnk\mu} \right|^2 \left[(1 + M^2_{x_P}) \hat{R}^P_{Wnk\mu} + M_{x_P} + M_{x_P} \left(\hat{R}^P_{Wnk\mu} \right)^2 \right] C_{m\mu} \tag{II.23}$$

This equation gives the sound power for blade passing frequency harmonics combined up to harmonic, N_h. The limits for k and μ are set by the number of cuton modes.

This equation can also be expressed as the sum of modal power levels, $\hat{\Pi}_{P_{nk\mu}}$, where:

$$\hat{\Pi}_P = \sum_{n=1}^{N_h} \sum_k \sum_\mu \hat{\Pi}_{P_{nk\mu}} \tag{II.24}$$

and:

$$\hat{\Pi}_{P_{nk\mu}} = \pi \frac{p_\infty}{\gamma_a p_P} \sqrt{\frac{T_P}{T_\infty}} \left| A^P_{Wnk\mu} \right|^2 \left[(1 + M^2_{x_P}) \hat{R}^P_{Wnk\mu} + M_{x_P} + M_{x_P} \left(\hat{R}^P_{Wnk\mu} \right)^2 \right] C_{m\mu} \tag{II.25}$$

This equation is equivalent to Equations (33) and (34). $\hat{R}^P_{Wnk\mu}$ is found in Equation (II.6) consistent with Equation (35), and $C_{m\mu}$ is found in Appendix III.

Thus, sound power level for a particular harmonic is:

$$\hat{\Pi}_{P_n} = \sum_k \sum_\mu \hat{\Pi}_{P_{nk\mu}} \tag{II.26}$$

where the summation is over propagating modes.

APPENDIX III: NORMALIZATION OF THE UNIFORM FLOW DUCT MODES

The purpose of this appendix is to evaluate the integral in Equation (II.17) analytically. Start with the expression for Bessel's equation for the radial direction. This equation is developed by applying the method of separation of variables to the convected wave equation in a mean axial flow. The equation related to the radial direction corresponds to Equation 4.4 in Reference 11.

$$\bar{r}\frac{d^2\psi(\kappa\bar{r})}{d\bar{r}^2} + \frac{d\psi(\kappa\bar{r})}{d\bar{r}} + \left(\kappa^2\bar{r} - \frac{m^2}{\bar{r}}\right)\psi(\kappa\bar{r}) = 0 \tag{III.1}$$

where κ is the separation constant and $\bar{r}$ is the dimensional radius. It will later be shown to also be the mode eigenvalue.

With the notation $\psi' = \frac{d\psi}{d\bar{r}}$:

$$\bar{r}\psi'' + \psi' + \left(\kappa^2\bar{r} - \frac{m^2}{\bar{r}}\right)\psi = 0 \tag{III.2}$$

The boundary conditions for this equation for a hardwalled annular duct are:

$$\psi'(\kappa r_d) = 0 \tag{III.3}$$

$$\psi'(\kappa r_h) = 0 \tag{III.4}$$

Multiply Equation (III.2) by $2\bar{r}\psi'(\kappa\bar{r})$:

$$2\bar{r}^2\psi''\psi' + 2\bar{r}(\psi')^2 + 2\left(\kappa^2\bar{r}^2 - m^2\right)\psi\psi' = 0 \tag{III.5}$$

Noting that $\left[(\bar{r}\psi')^2\right]' = 2\bar{r}^2\psi''\psi' + 2\bar{r}(\psi')^2$ and $\left(\psi^2\right)' = 2\psi\psi'$ Equation (III.5) becomes:

$$\left[(\bar{r}\psi')^2\right]' = -\left(\kappa^2\bar{r}^2 - m^2\right)\left(\psi^2\right)' \tag{III.6}$$

Integrate both sides of this equation across the duct:

$$\int_{r_h}^{r_d}\left[(\bar{r}\psi')^2\right]' d\bar{r} = -\int_{r_h}^{r_d}\left(\kappa^2\bar{r}^2 - m^2\right)\left(\psi^2\right)' d\bar{r} \tag{III.7}$$

or:

$$\left[\left\{\bar{r}_d\psi'(\kappa r_d)\right\}^2 - \left\{\bar{r}_h\psi'(\kappa r_h)\right\}^2\right] = -\int_{r_h}^{r_d}\left(\kappa^2\bar{r}^2 - m^2\right)\left(\psi^2\right)' d\bar{r} \tag{III.8}$$

By applying boundary conditions Equations (III.3) and (III.4) to Equation (III.8), the left hand side of this equation becomes zero and the ψ on the right hand side becomes the eigenfunction $\psi_{m\mu}\left(\kappa_{m\mu}r\right)$, within a constant. However, we do not apply the subscripts until the normalization is defined below. Therefore:

$$0 = -\int_{r_h}^{r_d} \left(\kappa^2 \bar{r}^2 - m^2\right)\left(\psi^2\right)' d\bar{r} \tag{III.9}$$

Integrating the right hand side by parts and rearranging:

$$\frac{1}{2}\left(\bar{r}^2 - \frac{m^2}{\kappa^2}\right)\psi^2 \Big|_{r_h}^{r_d} = \int_{r_h}^{r_d} \psi^2 \bar{r} d\bar{r} \tag{III.10}$$

or:

$$\frac{1}{2}\left[\left(r_d^2 - \frac{m^2}{\kappa^2}\right)\psi^2(\kappa r_d) - \left(r_h^2 - \frac{m^2}{\kappa^2}\right)\psi^2(\kappa r_h)\right] = \int_{r_h}^{r_d} \psi^2 \bar{r} d\bar{r} \tag{III.11}$$

Factor out r_d^2, normalize κ by r_d and normalize $\bar{r}$ by r_d. Also normalize ψ by ψ_{max} where $\psi_{\max}$ is the maximum value of each mode eigenfunction ($\psi_{\max} \equiv 1$ in CUP3D). Thus: $\kappa_{m\mu} = \kappa r_d$, $\sigma = \frac{r_h}{r_d}$ = hub/tip ratio of the duct, and $r = \frac{\bar{r}}{r_d}$ is the non-dimensional radius. Therefore:

$$\frac{1}{2}\left[\left(1 - \frac{m^2}{\kappa_{m\mu}^2}\right)\psi_{m\mu}^2(\kappa_{m\mu}) - \left(\sigma^2 - \frac{m^2}{\kappa_{m\mu}^2}\right)\psi_{m\mu}^2(\kappa_{m\mu}\sigma)\right] = \int_{\sigma}^{1} \psi_{m\mu}^2 r dr \tag{III.12}$$

The integral on the right hand side of this equation is the same as the left hand side of Equation (II.17). Thus $C_{m\mu}$ is:

$$C_{m\mu} = \frac{1}{2}\left[\left(1 - \frac{m^2}{\kappa_{m\mu}^2}\right)\psi_{m\mu}^2(\kappa_{m\mu}) - \left(\sigma^2 - \frac{m^2}{\kappa_{m\mu}^2}\right)\psi_{m\mu}^2(\kappa_{m\mu}\sigma)\right] \tag{III.13}$$

This equation applies to a locally constant area hardwall annular duct with locally uniform axial flow. $\kappa_{m\mu}$ are the mode eigenvalues and $\psi_{m\mu}$ are the mode eigenfunctions. Note that to calculate power only the mode eigenfunctions at the hub and tip locations are required.

APPENDIX IV: CALCULATING THE TOTAL ACOUSTIC FAR–FIELD PRESSURE

The purpose of this section is to derive the equations for the far–field directivity using the state vector solution (i.e. mode amplitudes) from CUP3D. CUP3D assumes that the system is formulated in the frequency domain where acoustic pressure is represented by the superposition of (m, μ) modes for each blade passing frequency harmonic. In polar coordinates of Figure 5 acoustic pressure can be expressed by:

$$P(R,\theta,\phi,t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} p_{nk\mu}(R,\theta) e^{i(m\phi - nB\Omega t)} \tag{IV.1}$$

where $p_{nk\mu}$ are the complex pressure mode amplitude coefficients obtained at the desired far–field points from a radiation code. Since negative harmonics are the complex conjugate of positive harmonics (i.e. $p_{-n-k\mu} = p^*_{nk\mu}$):

$$P(R,\theta,\phi,t) = 2\,\mathrm{Re}\left\{ \sum_{n=1}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} p_{nk\mu}(R,\theta) e^{i(m\phi - nB\Omega t)} \right\} \tag{IV.2}$$

Equation (IV.1) as applied to the far–field can also be expressed as the combination of pressure waves propagating from the inlet and nozzle. Thus:

$$p_{nk\mu}(R,\theta) = p_{Ink\mu}(R,\theta) + p_{Ank\mu}(R,\theta) \tag{IV.3}$$

where $p_{Ink\mu}(R,\theta)$ is the inlet complex pressure directivity for each (m, μ) mode, and $p_{Ank\mu}(R,\theta)$ is the aft complex pressure directivity for each (m,μ) mode. These complex pressure directivities can also be expressed as a superposition of state vector amplitudes, $A^P_{Wnk\mu}$, multiplied by directivity shape functions, $D_{nk\mu}$, at interface planes just upstream and just downstream of the noise source, for each radial mode order.

For inlet noise the interface plane, P, where the inlet is connected to the noise source is used (e.g. in Figure 7, $P = 1$) to obtain the upstream propagating pressure waves from the state vector, $A^P_{Wnk\mu}$ and the mode inlet directivity shape functions, $D_{IWnk\mu}(R,\theta)$. The CUP3D code is configured to permit multiple upstream pressure wave types so that these pressures are added together to obtain the complex pressure directivity for each (m, μ) mode, $p_{Ink\mu}(R,\theta)$. The number of forward propagating pressure wave types is given as N_{UPSTR}. Thus:

$$p_{Ink\mu}(R,\theta) = \sum_{W=1}^{N_{UPSTR}} A^P_{Wnk\mu} D_{IWnk\mu}(R,\theta) \tag{IV.4}$$

Note that for all modal representations developed to date (e.g. Reference 14) only one type of forward propagating mode has been found (i.e. $N_{UPSTR} = 1$). However, CUP3D uses Equation (IV.4) as shown to permit more upstream pressure wave types to be added in the future, if necessary.

For aft noise the interface plane where the aft radiation code is connected to the noise source is used (e.g. in Figure 7, $P = 3$) to obtain the downstream propagating pressure waves from the state vector,

$A^P_{Wnk\mu}$ and the mode aft directivity shape functions, $D_{AWnk\mu}(R,\theta)$. The CUP3D code is configured to permit multiple downstream pressure wave types so that these pressures are added together to obtain the complex pressure directivity for each *(m,μ)* mode, $p_{Ank\mu}(R,\theta)$. The number of propagating pressure wave types is assumed below to be $N_{PRESDIR}$ so that the aft pressure directivity for each *(m,μ)* mode becomes:

$$p_{Ank\mu}(R,\theta) = \sum_{W=N_{UPSTR}+1}^{N_{PRESDIR}} A^P_{Wnk\mu} D_{AWnk\mu}(R,\theta) \tag{IV.5}$$

For the modes presently in TFaNS and discussed in Section 3.2, $N_{PRESDIR} = 2$. However, CUP3D uses Equation (IV.5) as shown to permit more downstream pressure wave types to be added in the future, if necessary.

Thus, to calculate the far–field directivity the following items are needed for each pressure wave type:

1. The solution to the state vector, $A^P_{Wnk\mu}$, at the interface plane where a radiation acoustic element is connected to the noise source, inlet and aft.
2. Directivity shapes for each propagating pressure wave type from an inlet radiation calculation, $D_{IWnk\mu}(R,\theta)$ and an aft radiation calculation, $D_{AWnk\mu}(R,\theta)$. These are determined by inputting unit mode amplitudes into a radiation code and then calculating the far–field directivities.

Also, CUP3D calculates the following far–field pressure directivities:

- Total far–field sound pressure level (SPL) directivity (inlet and aft combined) for each harmonic
- Inlet SPL directivity for each harmonic
- Aft SPL directivity for each harmonic
- Inlet, aft and total SPL directivities for each circumferential mode order at each harmonic (summed over all radial mode orders)
- Inlet, aft and total SPL directivities for each circumferential and radial mode order at each harmonic

For this derivation we will concentrate on total directivities which can be represented as the combination of the inlet and aft directivities. Then more simplified forms of the equations will be derived. Given Equations (IV.1), (IV.3), (IV.4), and (IV.5), the acoustic pressure in the far–field at a given directivity angle in CUP3D is:

$$P(R,\theta,\phi,t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} p_{Ink\mu}(R,\theta)e^{i(m\phi - nB\Omega t)} + p_{Ank\mu}(R,\theta)e^{i(m\phi - nB\Omega t)} \tag{IV.6}$$

Or given that $m = nB - kV$:

$$P(R,\theta,\phi,t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} \left[p_{Ink\mu} + p_{Ank\mu}\right]e^{i[(nB-kV)\phi - nB\Omega t]} \tag{IV.7}$$

Let $\Theta_{nk}(\phi,t) = (nB - kV)\phi - nB\Omega t$ so that Equation (IV.7) becomes:

$$P(R,\theta,\phi,t) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{\mu=0}^{\infty} \left[p_{Ink\mu} + p_{Ank\mu}\right]e^{i\Theta_{nk}(\phi,t)} \tag{IV.8}$$

This equation now needs to be processed to obtain the root mean square acoustic pressure as follows:

$$P(R,\theta) = \sqrt{\overline{\overline{P^2(R,\theta,\phi,t)}}} \qquad \text{(IV.9)}$$

The notation implies the following operations: first square P, then time and circumferentially average P^2, and then take the square root. A circumferential average is being calculated because we rarely know the engine azimuthal angle relative to the location of engine circumferential origin. Thus:

$$P^2 = PP^* = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{\mu=0}^{\infty}\sum_{n'=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\sum_{\mu'=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]e^{i\Theta_{nk}(\phi,t)}\left[p^*_{In'k'\mu'} + p^*_{An'k'\mu'}\right]e^{-i\Theta_{n'k'}(\phi,t)} \qquad \text{(IV.10)}$$

Substituting back in for $\Theta_{nk}(\phi,t)$ and rearranging the exponent gives:

$$P^2 = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{\mu=0}^{\infty}\sum_{n'=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\sum_{\mu'=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\left[p^*_{In'k'\mu'} + p^*_{An'k'\mu'}\right]e^{i\{[(n-n')B-(k-k')V]\phi-(n-n')B\Omega t\}} \qquad \text{(IV.11)}$$

The magnitudes in the above equation can also be expressed as the summation over all μ's. Thus rearrange this equation to get:

$$P^2 = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{n'=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\}\left\{\sum_{\mu'=0}^{\infty}\left[p^*_{In'k'\mu'} + p^*_{An'k'\mu'}\right]\right\}e^{i\{[(n-n')B-(k-k')V]\phi-(n-n')B\Omega t\}} \qquad \text{(IV.12)}$$

Now average over time. It can be seen that when $n \neq n'$, then the acoustic pressure is sinusoidal and acoustic pressure over time averages to zero. Thus we get $\overline{e^{i\{n-n'\}B\Omega t}} = \delta_{nn'}$ and Equation (IV.12) becomes:

$$\overline{P^2(R,\theta,\phi,t)} = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\}\left\{\sum_{\mu'=0}^{\infty}\left[p^*_{Ink'\mu'} + p^*_{Ank'\mu'}\right]\right\}e^{-i[(k-k')V]\phi} \qquad \text{(IV.13)}$$

Let:

$$p_{nk} = \left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\} \qquad \text{(IV.14)}$$

and:

$$p^*_{nk'} = \left\{\sum_{\mu'=0}^{\infty}\left[p^*_{Ink'\mu'} + p^*_{Ank'\mu'}\right]\right\} \qquad \text{(IV.15)}$$

So that Equation (IV.13) becomes:

$$\overline{P^2(R,\theta,\phi,t)} = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty} p_{nk}p^*_{nk'}e^{-i\{(k-k')V\}\phi} \qquad \text{(IV.16)}$$

This can also be expressed as a one sided series on n where:

$$\overline{P^2(R,\theta,\phi,t)} = \sum_{n=1}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty} p_{nk}p^*_{nk'}e^{-i\{(k-k')V\}\phi} + p_{-nk}p^*_{-nk'}e^{-i\{(k-k')V\}\phi} \qquad \text{(IV.17)}$$

where we have dropped the n = 0 term. In the second term note that: $p^{*}_{-nk'} = p_{n-k'}$ and $p_{-nk} = p^{*}_{n-k}$ so that:

$$\overline{P^2(R,\theta,\phi,t)} = 2\sum_{n=1}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty} p_{nk}p^{*}_{nk'}e^{-i\{(k-k')V\}\phi} \tag{IV.18}$$

Equation (IV.18) shows that the root mean square acoustic field is spatially periodic in the ϕ (circumferential) direction with the same number of lobes as vanes. Substituting back Equations (IV.14) and (IV.15):

$$\overline{P^2(R,\theta,\phi,t)} = 2\sum_{n=1}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\}\left\{\sum_{\mu'=0}^{\infty}\left[p^{*}_{Ink'\mu'} + p^{*}_{Ank'\mu'}\right]\right\}e^{-i\{(k-k')V\}\phi} \tag{IV.19}$$

For our purposes we want the mean square pressure directivity for each harmonic separately so that:

$$\overline{P_n^2(R,\theta,\phi,t)} = 2\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\}\left\{\sum_{\mu'=0}^{\infty}\left[p^{*}_{Ink'\mu'} + p^{*}_{Ank'\mu'}\right]\right\}e^{-i\{(k-k')V\}\phi} \tag{IV.20}$$

The square root:

$$\sqrt{\overline{P_n^2(R,\theta,\phi,t)}} = \sqrt{2\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\}\left\{\sum_{\mu'=0}^{\infty}\left[p^{*}_{Ink'\mu'} + p^{*}_{Ank'\mu'}\right]\right\}e^{-i\{(k-k')V\}\phi}} \tag{IV.21}$$

is the general equation for the root mean square pressure directivity at an arbitrary circumferential angle. This equation is useful if the engine azimuthal angle is known relative to a known circumferential origin. In general the circumferential origin and the azimuthal angle may be known, but the effect of phase shifting in the noise measurement equipment is not known accurately. Thus, it is desirable to average Equation (IV.20) circumferentially. In Equation (IV.20), when k $\neq$ k', then the pressure is sinusoidal and averages to zero circumferentially. Therefore $\overline{e^{-i(k-k')V\phi}} = \delta_{kk'}$ so that:

$$\overline{\overline{P_n^2(R,\theta,\phi,t)}} = \overline{\overline{P_n P_n^{*}}} = 2\sum_{k=-\infty}^{\infty}\left\{\sum_{\mu=0}^{\infty}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right\}\left\{\sum_{\mu'=0}^{\infty}\left[p^{*}_{Ink\mu'} + p^{*}_{Ank\mu'}\right]\right\} \tag{IV.22}$$

Since the summation over μ is the complex conjugate of the summation over μ':

$$\overline{\overline{P_n^2(R,\theta,\phi,t)}} = \sum_{k=-\infty}^{\infty}\left|\sum_{\mu=0}^{\infty}\sqrt{2}\left[p_{Ink\mu} + p_{Ank\mu}\right]\right|^2 \tag{IV.23}$$

To get Equation (IV.9) take the square root of Equation (IV.23) and rearrange to get Equation (41) in Section 3.4.2:

$$P_n(R,\theta) = \sqrt{\sum_{k=-\infty}^{\infty}\left|\sum_{\mu=0}^{\infty}\sqrt{2}p_{Ink\mu} + \sum_{\mu=0}^{\infty}\sqrt{2}p_{Ank\mu}\right|^2} \tag{IV.24}$$

This gives us the root mean square of the total harmonic pressure pattern averaged circumferentially. Note that pressure as input to CUP3D is normalized by a reference pressure, p_∞. Since $\frac{P_{rms}}{p_\infty} = \sqrt{\overline{\overline{P_n^2}}}$, to get far−field SPL Levels:

$$SPL_{n_{total}} = 20\log\left(\frac{p_\infty P_n(R,\theta)}{P_{ref}}\right) \tag{IV.25}$$

where $P_{ref} = 20\mu\text{Pa}$.

Simplified versions of Equation (IV.24) are discussed in Section 3.4.2.

APPENDIX V: MODIFYING EVERSMAN INLET AND AFT RADIATION CODES TO INCORPORATE THEM INTO TFaNS

The Eversman Inlet and Aft Radiation Codes have been incorporated into TFaNS to calculate far–field noise and provide internal reflections from inlets and nozzles. The purpose of this appendix is to discuss the modifications to these codes and to derive the equations used in the modified codes. Modifications to the radiation codes include:

- Capability to run multiple modes and harmonics with unit input.
- Dimensional input format based on standard geometry and performance parameters.
- An Acoustic Properties File output for use by CUP3D.

In the sections which follow, these modifications will be further discussed.

V.1 MULTIPLE MODE RUNNING & CHANGES TO THE INPUT FORMAT

The capability to run multiple modes and harmonics was added by creating a mode search routine. This routine decides which mode and frequencies to run through a radiation module and then runs the module with unit input for each (m,μ) mode.

As part of the creation of TFaNS versions of the inlet and aft radiation modules, the input file format was changed to be more consistent with standard design parameters. Two parameters are now calculated instead of being input: The reduced frequency, ETAR (η_r) and the circumferential mode order, m. The reduced frequency is defined by:

$$\eta_r = \frac{\omega r_d}{c_\infty} \tag{V.1}$$

Since $\omega = nB\Omega$, $c_\infty = \sqrt{\gamma_a R_a T_\infty}$, and $\Omega = \frac{2\pi N_1}{60}$:

$$\eta_r = nB\frac{\pi N_1 r_d}{360\sqrt{\gamma_a R_a T_\infty}} \tag{V.2}$$

where:

N_1 = *fan rotational speed (rpm)*
r_d = *fan duct radius (inches)*
R_a = *Ideal gas constant for air = 1716.5* $ft^2/sec^2\ {}^0R$
γ_a = *ratio of specific heats = 1.4 for air*

In an adiabatic flow the total temperature and static temperature can be related to the axial Mach number far from the nacelle as:

$$\frac{T_{0_1}}{T_\infty} = 1 + \frac{\gamma_a - 1}{2}M_{x\infty}^2 \tag{V.3}$$

Therefore substituting Equation (V.3) into Equation (V.2):

$$\eta_r = nB\frac{\pi N_1 r_d}{360\sqrt{\gamma_a R_a T_{0_1}}}\left(1 + \frac{\gamma_a - 1}{2}M_{x\infty}^2\right)^{1/2} \tag{V.4}$$

A common design parameter is the corrected fan rotational speed, N_{1c}:

$$N_{1c} = \frac{N_1}{\sqrt{\frac{T_{0_1}}{518.67}}} \tag{V.5}$$

Substituting Equation (V.5) into Equation (V.4):

$$\eta_r = nBM_{tip_c} \tag{V.6}$$

where:

$$M_{tip_c} = \frac{\pi N_{1c} r_d}{360\sqrt{\gamma_a R_a (518.67)}}\left(1 + \frac{\gamma_a - 1}{2} M_{x\infty}^2\right)^{1/2} \tag{V.7}$$

Equations (V.6) and (V.7) are used by both the inlet and aft radiation codes. N_{1c}, r_d and B are input by the user. $M_{x\infty}$ is the Mach number far from the nacelle and is available from the radiation code mesh generation output file.

Also calculated by the TFaNS radiation modules is the circumferential mode order, $m = nB - kV$. This is accomplished by having the user input the number of stator vanes, V into the code. The mode search routine then determines the modes it needs to calculate by varying k and μ for a user specified number of BPF harmonics, n.

V.2 MODIFYING RADIATION CODES TO OUTPUT ACOUSTIC PROPERTIES FILES

To correctly output information to an Acoustic Properties File, two major items must be output from the radiation codes:

- Scattering Coefficients
- Far–field directivities

A method for creating this output is discussed below.

V.2.1 Scattering Coefficients

Both the inlet and aft radiation codes are formulated from the same theory using the same basis for a coordinate system and exponential notation (see Reference 3). The inlet and aft radiation codes are formulated in terms of the perturbation velocity potential. Acoustic pressure can then be calculated from this velocity potential. For a single acoustic mode at the radiation code input plane, the perturbation velocity potential, $\tilde{\varphi}$, is given by Equation 34, Reference 3:

$$\tilde{\varphi}^{P}_{Wnk\mu}(X,r,\phi,t) \sim \psi^{P}_{Wm\mu}(r)e^{i\left(\omega t - m\phi - k^{P}_{xWnk\mu}X\right)} \tag{V.8}$$

where the notation has been made consistent with this report except for k_x which is the axial wavenumber in the Eversman radiation codes. Also, an equal sign has not been used here because in the actual Eversman codes a perturbation velocity potential mode amplitude, φ, is also included for each mode but does appear in Equation 34, Reference 3. Thus, including the perturbation velocity potential mode amplitude in the above equation:

$$\tilde{\varphi}^{P}_{Wnk\mu}(X,r,\phi,t) = \varphi^{P}_{Wnk\mu}\psi^{P}_{Wm\mu}(r)e^{i\left(\omega t - m\phi - k^{P}_{xWnk\mu}X\right)} \tag{V.9}$$

The perturbation potential relates to the acoustic pressure through Equation 16 of Reference 3:

$$p^P_{Wnk\mu_{rad}}(X,r,\phi,t) = -\rho_f\left[\frac{\partial\tilde{\varphi}}{\partial t} + \left(\vec{\nabla}\varphi_f\cdot\vec{\nabla}\tilde{\varphi}\right)\right] \tag{V.10}$$

where $p^P_{Wnk\mu_{rad}}$ is the modal acoustic pressure in the Eversman radiation codes.

Substituting Equation (V.9) into Equation (V.10) results in Equation 122 in Reference 3. A form of this equation which includes the exponential is:

$$p^P_{Wnk\mu_{rad}}(X,r,\phi,t) = -i\rho_f\omega\left[1 - \frac{U_f k^P_{xWnk\mu}}{\omega}\right]\varphi^P_{Wm\mu}\psi^P_{Wnk\mu}(r)e^{i\left[\omega t - m\phi - k^P_{xWnk\mu}X\right]} \tag{V.11}$$

Non-dimensionalize this equation by r_d and c_∞ to get:

$$p^P_{Wnk\mu_{rad}}(x,r,\phi,t) = -i\frac{\rho_f c_\infty}{r_d}\eta_r\left[1 - \frac{M_f\hat{k}^P_{xWnk\mu}}{\eta_r}\right]\varphi^P_{Wnk\mu}\psi^P_{Wm\mu}(r)e^{i\left[nB\Omega t - m\phi - \hat{k}^P_{xWnk\mu}x\right]} \tag{V.12}$$

Note that the perturbation velocity potential mode amplitude, φ, the local static density, ρ_f, the duct radius, r_d, and far-field acoustic speed, c_∞ are dimensional so that $p^P_{Wnk\mu_{rad}}$ is dimensional. Compare this expression to Equation (20) which is the expression for the acoustic pressure in CUP3D. For a single mode Equation (20) is written as:

$$p^P_{Wnk\mu}(x,r,\phi,t) = p_\infty A^P_{Wnk\mu}\psi^P_{Wm\mu}(r)e^{i\left[m\phi + \tilde{\gamma}^P_{Wnk\mu}\left(x - x^P_W\right) - nB\Omega t\right]} \tag{V.13}$$

Comparing Equation (V.12) to Equation (V.13) it is observed that:

- The axial origin of the Eversman Radiation codes may be different from that of CUP3D.
- One equation appears to be the complex conjugate of the other, i.e. $\left\{p^P_{Wnk\mu_{rad}}\right\}^* = p^P_{Wnk\mu}$.

Regarding the axial origin, in CUP3D the axial location of the origin is given explicitly as the interface plane so that at the interface plane, x^P_W, $x - x^P_W = 0$. In the Eversman radiation codes, the axial origin at the input plane is set to be at the interface plane, x^P_W, so that $x = x^P_W = 0$. As a result, the axial locations are forced to be the same for CUP3D and the radiation codes.

Now we check the complex conjugate observation by taking the complex conjugate of Equation (V.12):

$$\left\{p^P_{Wnk\mu_{rad}}\right\}^* = i\frac{\rho_f c_\infty}{r_d}\eta_r\left[1 - \frac{M_f\hat{k}^P_{xWnk\mu}}{\eta_r}\right]^*\left\{\varphi^P_{Wnk\mu}\psi^P_{Wm\mu}(r)\right\}^* e^{i\left[-nB\Omega t + m\phi + \left(\hat{k}^P_{xWnk\mu}\right)^* x\right]} \tag{V.14}$$

Note that the complex conjugate observation above is not true for cutoff modes. Instead, we must compare Equation (V.13) with Equation (V.14). These two equations cannot be considered the equal to each other unless $\tilde{\gamma}^{P}_{W_{nk\mu}} = \left(\hat{k}^{P}_{xW_{nk\mu}}\right)^{*}$. With some analysis (which will not be covered here), this expression has been found to be true for both the Eversman inlet and aft radiation codes. In the case of the inlet radiation code, a difference in the definition of positive x and M_f complicates this result slightly, but does not impact on the equivalence of the two equations.

For consistency, make the expression in the exponential common between CUP3D and the radiation codes. Also, ψ is a real number. Thus:

$$p^{P}_{W_{nk\mu}} = \left\{p^{P}_{W_{nk\mu_{rad}}}\right\}^{*} = i\frac{\rho_f c_\infty}{r_d}\eta_r\left[1 - \frac{M_f\hat{k}^{P}_{xW_{nk\mu}}}{\eta_r}\right]^{*}\left\{\varphi^{P}_{W_{nk\mu}}(r)\right\}^{*}\psi^{P}_{W_{m\mu}}(r)e^{i\left[-nB\Omega t+m\phi+\tilde{\gamma}^{P}_{W_{nk\mu}}x\right]} \tag{V.15}$$

By setting the amplitudes of Equation (V.13) equal those of Equation (V.15) and solving for $A^{P}_{W_{nk\mu}}$:

$$A^{P}_{W_{nk\mu}} = i\frac{\rho_f c_\infty}{p_\infty r_d}\eta_r\left[1 - \frac{M_f\hat{k}^{P}_{xW_{nk\mu}}}{\eta_r}\right]^{*}\left\{\varphi^{P}_{W_{nk\mu}}(r)\right\}^{*} \tag{V.16}$$

Thus this expression for $A^{P}_{W_{nk\mu}}$ is used to calculate the scattering coefficients for the radiation codes.

V.2.1.1 Inlet Radiation Code Scattering Coefficients

Referring to the scattering coefficients for the inlet radiation code in Section I.3.2 of Appendix I, we note that only the S^{DD}_{21} scattering coefficient is needed where D refers to the interface plane where the noise source and the inlet meet (e.g. in Figure 7, $D = 1$).

Equation (22) shows how scattering coefficients may be calculated for the inlet radiation code. Thus, knowing that radiation codes only scatter on μ, and assuming a unit amplitude pressure wave input (i.e. $A^{D}_{1\,nk\mu} = 1.0 + i0.0$):

$$S^{DD}_{21\,nk\mu';nk\mu} = \frac{A^{D}_{2\,nk\mu'}}{A^{D}_{1\,nk\mu}} \tag{V.17}$$

Substituting Equation (V.16) into Equation (V.17):

$$S^{DD}_{21\,nk\mu';nk\mu} = \frac{\left[1 - \frac{M_f\hat{k}^{D}_{x2\,nk\mu'}}{\eta_r}\right]^{*}\left\{\varphi^{D}_{2\,nk\mu'}\right\}^{*}}{\left[1 - \frac{M_f\hat{k}^{D}_{x1\,nk\mu}}{\eta_r}\right]^{*}\left\{\varphi^{D}_{1\,nk\mu}\right\}^{*}} \tag{V.18}$$

Since this is a ratio, non−dimensional perturbation velocity potentials can be used in this equation. Equation (V.18) is the expression used to calculate scattering coefficients in the inlet radiation code.

V.2.1.2 Aft Radiation Code Scattering Coefficients

Referring to the scattering coefficients for the aft radiation code in Section I.3.5 of Appendix I, note that only the S_{12}^{UU} scattering coefficient is needed where U refers to the interface plane where the noise source and the nozzle meet (e.g. in Figure 7, $U = 3$).

Equation (25) shows how scattering coefficients may be calculated for the aft radiation code. Thus, knowing that radiation codes only scatter on μ, and assuming a unit amplitude pressure wave input (i.e. $A_{2\,nk\mu}^{U} = 1.0 + i0.0$):

$$S_{12\;nk\mu';nk\mu}^{UU} = \frac{A_{1\,nk\mu'}^{U}}{A_{2\,nk\mu}^{U}} \tag{V.19}$$

Substituting Equation (V.16) into Equation (V.19):

$$S_{12\;nk\mu';nk\mu}^{UU} = \frac{\left[1 - \dfrac{M_j \hat{k}_{x1\,nk\mu'}^{U}}{\eta_r}\right]^{*} \left\{\varphi_{1\,nk\mu'}^{U}\right\}^{*}}{\left[1 - \dfrac{M_j \hat{k}_{x2\,nk\mu}^{U}}{\eta_r}\right]^{*} \left\{\varphi_{2\,nk\mu}^{U}\right\}^{*}} \tag{V.20}$$

V.2.2 Radiation Pressure Directivities

To correctly output far-field directivities from the Eversman codes, examine Equation 83 in Reference 3. Putting this equation into the notation of this report:

$$\tilde{\varphi}_{Wnk\mu}^{P}(x,r,\phi,t) = \overline{\varphi}_{Wnk\mu}^{P}(x,r)e^{i(\omega t - m\phi)} \tag{V.21}$$

We can convert the coordinate system from cylindrical to spherical coordinates and change notation so that:

$$\tilde{\varphi}_{Wnk\mu}^{P}(R,\theta,\phi,t) = \overline{\varphi}_{Wnk\mu}^{P}(R,\theta)e^{i(\omega t - m\phi)} \tag{V.22}$$

The acoustic pressure for a given mode can be written in the same form:

$$P_{nk\mu_{rad}}(R,\theta,\phi,t) = p_{nk\mu_{rad}}(R,\theta)e^{i(\omega t - m\phi)} \tag{V.23}$$

When comparing this equation to Equation (IV.1) in Appendix IV we observe that:

$$p_{nk\mu}(R,\theta) = \left\{p_{nk\mu_{rad}}(R,\theta)\right\}^{*} \tag{V.24}$$

For a given wave type, Equations (IV.4) or (IV.5) give the directivity calculation. In the Eversman radiation codes where there is only one upstream going pressure wave and one downstream going pressure wave:

$$p_{nk\mu}(R,\theta) = A_{Wnk\mu}^{P} D_{nk\mu}(R,\theta) \tag{V.25}$$

So that in the radiation codes:

$$p_{nk\mu_{rad}}(R,\theta) = A^P_{Wnk\mu_{rad}} D_{nk\mu_{rad}}(R,\theta) \tag{V.26}$$

or:

$$p_{nk\mu}(R,\theta) = \left[A^P_{Wnk\mu_{rad}}\right]^* \left[D_{nk\mu_{rad}}(R,\theta)\right]^* \tag{V.27}$$

When the radiation codes are run, the input value is $A^P_{Wnk\mu_{rad}} = 1.0 + i0.0$. Thus for this case:

$$\left\{A^P_{Wnk\mu_{rad}}\right\}^* = A^P_{Wnk\mu} \tag{V.28}$$

Since $A^P_{Wnk\mu}$ is calculated by CUP3D after coupling, the radiation codes only need to output the directivity shapes based on the above unit input. Therefore:

$$DIRECTIN(n,k,\mu,IANGLE) = \left\{D_{nk\mu_{rad}}(R,\theta)\right\}^* \tag{V.29}$$

where DIRECTIN may be found in Appendix I as an Acoustics Properties File parameter for inlet and aft radiation codes.

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13. ABSTRACT ***(Maximum 200 words)***

TFaNS is the Tone Fan Noise Design/Prediction System developed by Pratt & Whitney under contract to NASA Lewis (presently NASA Glenn). The purpose of this system is to predict tone noise emanating from a fan stage including the effects of reflection and transmission by the rotor and stator and by the duct inlet and nozzle. These effects have been added to an existing annular duct/isolated stator noise prediction capability. TFaNS consists of: The codes that compute the acoustic properties (reflection and transmission coefficients) of the various elements and write them to files. CUP3D: Fan Noise Coupling Code that reads these files, solves the coupling problem, and outputs the desired noise predictions. AWAKEN: CFD/Measured Wake Postprocessor which reformats CFD wake predictions and/or measured wake data so it can be used by the system. This volume of the report provides technical background for TFaNS including the organization of the system and CUP3D technical documentation. This document also provides information for code developers who must write Acoustic Property Files in the CUP3D format. This report is divided into three volumes: Volume I: System Description, CUP3D Technical Documentation, and Manual for Code Developers; Volume II: User's Manual, TFaNS Vers. 1.4; Volume III: Evaluation of System Codes.

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